Problem

Find all ordered pairs, \((a, b)\), that satisfy \(\frac{a-b}{a+b} = 9\) and \(\frac{ab}{a+b} = -60\).

Solution

Multiplying both sides of the first equation, \(\frac{a-b}{a+b} = 9\), by \(a+b\) gives \(a-b = 9a + 9b\) and so \(-8a = 10b\) or \(-4a = 5b\). Thus, \(a = -\frac{5}{4}b\).

Multiplying both sides of the second equation, \(\frac{ab}{a+b} = -60\), by \(a+b\) gives \(ab = -60a - 60b\). Substituting \(a = -\frac{5}{4}b\) into \(ab = -60a - 60b\), we get

\[
ab = -60a - 60b
\]

\[
\left(-\frac{5}{4}b\right) (b) = -60 \left(-\frac{5}{4}b\right) - 60b
\]

\[-\frac{5}{4}b^2 = 75b - 60b
\]

\[-\frac{5}{4}b^2 = 15b
\]

\[b^2 = -12b
\]

\[b^2 + 12b = 0
\]

Notice that \(b = 0\) satisfies this equation. Thus \(b = 0\) is one possibility. When \(b \neq 0\), we can divide both sides of the equation by \(b\) to get \(b + 12 = 0\), or \(b = -12\). Thus, \(b = 0\) or \(b = -12\).

If \(b = 0\), then \(a = -\frac{5}{4}(0) = 0\). But this gives us a denominator of 0 in each of the original equations. Therefore, \(b \neq 0\).

If \(b = -12\), then \(a = -\frac{5}{4}(-12) = 15\).

Therefore, the only ordered pair that satisfies both equations is \((15, -12)\).