(a, b)

Problem of the Week
Problem D and Solution
Fraction Distraction

Problem

Find all ordered pairs, (a, b), that satisfy $\dfrac{a-b}{a+b} = 9$ and $\dfrac{ab}{a+b} = -60$.

Solution

Multiplying both sides of the first equation, $\dfrac{a-b}{a+b} = 9$, by $a+b$ gives $a - b = 9a + 9b$ and so $-8a = 10b$ or $-4a = 5b$. Thus, $a = -\dfrac{5}{4}b$.

Multiplying both sides of the second equation, $\dfrac{ab}{a+b} = -60$, by $a+b$ gives $ab = -60a - 60b$.

Substituting $a = -\dfrac{5}{4}b$ into $ab = -60a - 60b$, we get

$$\begin{aligned} ab &= -60a - 60b \\ \left(-\frac{5}{4}b\right)(b) &= -60\left(-\frac{5}{4}b\right) - 60b \\ -\frac{5}{4}b^2 &= 75b - 60b \\ -\frac{5}{4}b^2 &= 15b \\ b^2 &= -12b \\ b^2 + 12b &= 0 \end{aligned}$$

Notice that $b = 0$ satisfies this equation. Thus $b = 0$ is one possibility. When $b \neq 0$, we can divide both sides of the equation by b to get $b + 12 = 0$, or $b = -12$. Thus, $b = 0$ or $b = -12$.

If $b = 0$, then $a = -\dfrac{5}{4}(0) = 0$. But this gives us a denominator of 0 in each of the original equations. Therefore, $b \neq 0$.

If $b = -12$, then $a = -\dfrac{5}{4}(-12) = 15$.

Therefore, the only ordered pair that satisfies both equations is $(15, -12)$.