Problem of the Week
Problem D and Solution
Two Equations and Two Variables

Problem
If \(2x = 3y + 11\) and \(2^x = 2^{4(y+1)}\), determine the value of \(x + y\).

Solution
Solution 1
Since \(2^x = 2^{4(y+1)}\), it follows that \(x = 4(y+1)\), or \(x = 4y + 4\). We now have the following two equations.

\[
\begin{align*}
2x &= 3y + 11 \quad \text{(1)} \\
x &= 4y + 4 \quad \text{(2)}
\end{align*}
\]

We can substitute equation (2) into equation (1) for \(x\).

\[
\begin{align*}
2x &= 3y + 11 \\
2(4y + 4) &= 3y + 11 \\
8y + 8 &= 3y + 11 \\
5y &= 3 \\
y &= \frac{3}{5}
\end{align*}
\]

Now, we can substitute \(y = \frac{3}{5}\) into equation (2) to solve for \(x\).

\[
\begin{align*}
x &= 4y + 4 \\
 &= 4 \left( \frac{3}{5} \right) + 4 \\
 &= \frac{12}{5} + \frac{20}{5} \\
 &= \frac{32}{5}
\end{align*}
\]

Now that we have the values of \(x\) and \(y\), we can determine the value of \(x + y\).

\[
x + y = \frac{32}{5} + \frac{3}{5} = \frac{35}{5} = 7
\]

Therefore, the value of \(x + y\) is 7.

Solution 2
We can solve this problem in a faster way without finding the values of \(x\) and \(y\). Since \(2^x = 2^{4(y+1)}\), it follows that \(x = 4(y+1)\), or \(x = 4y + 4\). We now have the following two equations.

\[
\begin{align*}
2x &= 3y + 11 \quad \text{(1)} \\
x &= 4y + 4 \quad \text{(2)}
\end{align*}
\]

We can subtract equation (2) from equation (1), and obtain the equation \(x = -y + 7\). Rearranging this equation gives \(x + y = 7\). Therefore, the value of \(x + y\) is 7.