Problem of the Week

Problem C and Solution

Altitude Change

Problem

In acute $\triangle ABC$, two altitudes have been drawn in. Point $M$ lies on $AB$ so that $CM$ is an altitude of $\triangle ABC$, and point $N$ lies on $AC$ so that $BN$ is an altitude of $\triangle ABC$.

Suppose $CM = 32$ cm, $AB = 36$ cm, and $AC = 40$ cm. Determine the length of altitude $BN$.

Solution

The area of a triangle is determined using the formula

$$\text{area} = \frac{\text{base} \times \text{height}}{2}$$

The height of the triangle is the length of an altitude and the base of the triangle is the length of the side to which a particular altitude is drawn.

Thus,

$$\begin{align*}
\text{Area } \triangle ABC &= \frac{AB \times CM}{2} \\
&= \frac{36 \times 32}{2} \\
&= 576 \text{ cm}^2
\end{align*}$$

Also,

$$\begin{align*}
\text{Area } \triangle ABC &= \frac{AC \times BN}{2} \\
576 &= \frac{40 \times BN}{2} \\
1152 &= 40 \times BN \\
BN &= 28.8 \text{ cm}
\end{align*}$$

Therefore, the length of altitude $BN$ is 28.8 cm.