



112

2021

## Problem of the Week

### Problem C and Solution

#### Chalking it Up

##### Problem

Marta used chalk to create a sequence of six numbers on the sidewalk outside her apartment building. After the first two numbers, each number in the sequence equals the sum of the previous two numbers. Marta started with the number 112 and ended with the number 2021. What are the other four numbers in her sequence?

##### Solution

Let  $a$  represent the second number in the sequence.

Since the third number is the sum of the previous two numbers, the third number is  $112 + a$ .

Since the fourth number is the sum of the previous two numbers, the fourth number is  $a + (112 + a) = 112 + 2a$ .

Since the fifth number is the sum of the previous two numbers, the fifth number is  $(112 + a) + (112 + 2a) = 224 + 3a$ .

Since the sixth number is the sum of the previous two numbers, the sixth number is  $(112 + 2a) + (224 + 3a) = 336 + 5a$ . But the sixth number in the sequence is 2021. Therefore,

$$336 + 5a = 2021$$

$$336 + 5a - 336 = 2021 - 336$$

$$5a = 1685$$

$$\frac{5a}{5} = \frac{1685}{5}$$

$$a = 337$$

We now know that the second number is 337, so we can determine the remaining numbers in the sequence by substituting into the expressions above or by simply using the rule to generate the remaining numbers. Using the rule, the third number is  $112 + 337 = 449$ , the fourth number is  $337 + 449 = 786$ , and the fifth number is  $449 + 786 = 1235$ . As a check, we can use the rule to determine the sixth number, obtaining  $786 + 1235 = 2021$ , as required.

Therefore, the other four numbers in Marta's sequence are 337, 449, 786, and 1235.