Problem of the Week
Problem D and Solution
Add On!

Problem

When sixty *consecutive odd* integers are added together, their sum is 4800. Determine the largest of the sixty integers.

Solution

Solution 1

In this solution we will solve using patterns.

Let a represent the smallest number. Since the numbers are odd, they increase by 2. So the second number is $(a + 2)$, the third is $(a + 4)$, the fourth is $(a + 6)$, and so on. What does the sixtieth number look like?

A closer look at the numbers reveals that the second number is $(a + 1(2))$, the third is $(a + 2(2))$, the fourth is $(a + 3(2))$, and so on. Following the pattern, the sixtieth number is $(a + 59(2)) = a + 118$. Then

$$\begin{aligned}
a + (a + 2) + (a + 4) + (a + 6) + \cdots + (a + 118) &= 4800 \\
60a + 2 + 4 + 6 + \cdots + 118 &= 4800 \\
60a + 2(1 + 2 + 3 + \cdots + 59) &= 4800 \\
60a + 2\left(\frac{59 \times 60}{2}\right) &= 4800, \text{ using the helpful formula.} \\
60a + 3540 &= 4800 \\
60a &= 1260 \\
a &= 21 \\
a + 118 &= 139
\end{aligned}$$

Therefore, the largest odd integer in the sum is 139.

Solution 2

In this solution we will use averages to solve the problem.

Let A represent the average of the sixty consecutive odd integers. The average times the number of integers equals the sum of the integers. Since the sum of the sixty integers is 4800, then $60A = 4800$ and $A = 80$.

Now the integers in the sequence are consecutive odd integers. The average is even. It follows that 30 integers are below the average and 30 integers are above. We are looking for the 30^{th} odd integer after the average. In fact we want the 30^{th} odd integer after the odd number 79, the first odd integer below the average. This integer is easily found,

$$79 + 30(2) = 79 + 60 = 139.$$

Therefore, the largest odd integer in the sum is 139.

Solution 3

In this solution we will use arithmetic sequences. This solution is presented last since many students in grade 9 or 10 have not encountered arithmetic sequences yet.

An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. The general term, t_n, of an arithmetic sequence is $t_n = a + (n-1)d$, where a is the first term, d is the difference between consecutive terms, and n is the term number. The sum of the first n terms of an arithmetic sequence, S_n, can be found using the formula $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$, where $a,\ d,$ and n are the same variables used in the general term formula.

Let a represent the first term in the sequence. Since the integers in the sequence are consecutive and odd, the integers go up by two. Therefore, $d = 2$. Since there are 60 terms in the sequence, $n = 60$. The sum of the sixty integers in the sequence is 4800, so $S_{60} = 4800$.

$$\begin{array}{rrcl} & S_n &=& \dfrac{n}{2}\left(2a + (n-1)d\right) \\ & 4800 &=& \dfrac{60}{2}\left(2a + (60-1)(2)\right) \\ & 4800 &=& 30(2a + 59(2)) \\ \text{Dividing by 30,} & 160 &=& 2a + 118 \\ & 42 &=& 2a \\ & 21 &=& a \end{array}$$

Since we want the largest integer in the sequence, we are looking for the sixtieth term.

$$\begin{array}{rrcl} \text{Using} & t_n &=& a + (n-1)d, \quad \text{with } a = 21, d = 2, n = 60 \\ & t_{60} &=& 21 + 59(2) \\ & t_{60} &=& 139 \end{array}$$

Therefore, the largest odd integer in the sum is 139.