



Problem of the Week

Problem D and Solution

Sibling Rivalry

Problem

Akira and Hideo are twins with different jobs. Akira earns five-eighths of what Hideo earns, but Akira's expenses are half of Hideo's. Akira ends up saving 40% of his income. What percentage of his income does Hideo save?

Solution

Solution 1: Using only one variable

Let h represent Hideo's income. Then Akira's income is $\frac{5}{8}h$.

Since Akira saves 40% of his income, his expenses are $100\% - 40\% = 60\%$ of his income. Therefore, Akira's expenses are $60\% \times \frac{5}{8}h = \frac{60}{100} \times \frac{5}{8}h = \frac{3}{8}h$.

Akira's expenses are one-half of Hideo's expenses so Hideo's expenses are twice Akira's expenses. Therefore, Hideo's expenses are $2 \times \frac{3}{8}h = \frac{3}{4}h = 0.75h = 75\%$ of h . Since Hideo's expenses are 75% of his income, he saves $100\% - 75\% = 25\%$ of his income.

Therefore, Hideo saves 25% of his income.

Solution 2: Using two variables

Let x represent Hideo's income and y represent Hideo's expenses. Then Akira's income is $\frac{5}{8}x$ and his expenses are $\frac{1}{2}y$.

Since Akira saves 40% of his income, his expenses are 60% of his income.

$$\begin{aligned}\frac{1}{2}y &= 0.60 \times \frac{5}{8}x \\ \frac{1}{2}y &= \frac{6}{10} \times \frac{5}{8}x \\ \frac{1}{2}y &= \frac{3}{8}x \\ y &= \frac{3}{4}x\end{aligned}$$

Hideo saves whatever is left of his income after expenses. Therefore Hideo saves

$$x - y = x - \frac{3}{4}x = \frac{1}{4}x = 0.25x = 25\% \text{ of } x.$$

Therefore, Hideo saves 25% of his income.

**Solution 3: Using two variables a bit differently**

Let $8x$ represent Hideo's income and $2y$ represent Hideo's expenses. Then Akira's income is $\frac{5}{8}(8x) = 5x$ and his expenses are $\frac{1}{2}(2y) = y$.

Since Akira saves 40% of his income, his expenses are 60% of his income.

$$y = 0.60 \times 5x$$

$$y = \frac{6}{10} \times 5x$$

$$y = 3x$$

Hideo earns $8x$ and his expenses are $2y$ so his savings are $8x - 2y$. We want the ratio of his savings to his income, $\frac{8x - 2y}{8x} = \frac{8x - 2(3x)}{8x} = \frac{2x}{8x} = \frac{1}{4}$ or 25% .

Therefore, Hideo saves 25% of his income.