Problem of the Week
Problem D and Solution
One, Two, Tree!

Problem

Every year on November 1, the Stablo family plants a tree. This tradition has happened for generations and the first tree the family planted is now 183 years old. Two of the trees were planted in their backyard; one is an apple tree and the other is a maple tree. The maple tree is older than 100, but was not the first tree planted. On November 1 of this year, the age of the maple tree is fifteen times the age of the apple tree.

On November 1 of some year from now, the age of the maple tree will be eleven times the age of the apple tree. How old are the trees that year?

Solution
Solution 1:

Since the age of the maple tree is fifteen times the age of the apple tree, the maple tree's age must be a multiple of 15 that is greater than 100 and less than 183. The maple tree's possible ages and corresponding possible ages for the apple tree are shown in the table below.

Maple Tree's Age	105	120	135	150	165	180
Apple Tree's Age	7	8	9	10	11	12

At some time in the future, the age of the maple tree will be eleven times the age of the apple tree. Let's let n be the number of years from now until this is true, where n is an integer.

We will look at the possible cases.

1. Could the apple tree be 7 years old and the maple tree 105 years old now?

   Then in n years, the apple tree would be $7 + n$ years old and the maple tree would be $105 + n$ years old. If in n years the age of the maple tree is eleven times the age of the apple tree, then we have $11(7 + n) = 105 + n$. Solving for n, we get $77 + 11n = 105 + n$ or $10n = 28$ or $n = 2.8$. Since n is not an integer, it is not possible for the apple tree to be 7 and the maple tree to be 105.

2. Could the apple tree be 8 years old and the maple tree 120 years old now?

   Then in n years, the apple tree would be $8 + n$ years old and the maple tree would be $120 + n$ years old. If in n years the age of the maple tree is eleven times the age of the apple tree, then we have $11(8 + n) = 120 + n$. Solving for n, we get $88 + 11n = 120 + n$ or $10n = 32$ or $n = 3.2$. Since n is not an integer, it is not possible for the apple tree to be 8 and the maple tree to be 120.

3. Could the apple tree be 9 years old and the maple tree 135 years old now?

   Then in n years, the apple tree would be $9 + n$ years old and the maple tree would be $135 + n$ years old. If in n years the age of the maple tree is eleven times the age of the apple tree, then we have $11(9 + n) = 135 + n$. Solving for n, we get $99 + 11n = 135 + n$ or $10n = 36$ or $n = 3.6$. Since n is not an integer, it is not possible for the apple tree to be 9 and the maple tree to be 135.

4. Could the apple tree be 10 years old and the maple tree 150 years old now?

   Then in n years, the apple tree would be $10 + n$ years old and the maple tree would be $150 + n$ years old. If in n years the age of the maple tree is eleven times the age of the apple tree, then we have $11(10 + n) = 150 + n$. Solving for n, we get $110 + 11n = 150 + n$ or $10n = 40$ or $n = 4$. This is 4 years from now. The apple tree would then be 14 and the maple tree would be 154. Since $14 \times 11 = 154$, at this point, the maple tree's age is eleven times the apple tree's age. So this is a solution, however we need to check the two remaining possibilities for completeness.

5. Could the apple tree be 11 years old and the maple tree 165 years old now?

   Then in n years, the apple tree would be $11 + n$ years old and the maple tree would be $165 + n$ years old. If in n years the age of the maple tree is eleven times the age of the apple tree, then we have $11(11 + n) = 165 + n$. Solving for n, we get $121 + 11n = 165 + n$ or $10n = 44$ or $n = 4.4$. Since n is not an integer, it is not possible for the apple tree to be 11 and the maple tree to be 165.

6. Could the apple tree be 12 years old and the maple tree be 180 years old now?

   Then in n years, the apple tree would be $12 + n$ years old and the maple tree would be $180 + n$ years old. If in n years the age of the maple tree is eleven times the age of the apple tree, then we have $11(12 + n) = 180 + n$. Solving for n, we get $132 + 11n = 180 + n$ or $10n = 48$ or $n = 4.8$. Since n is not an integer, it is not possible for the apple tree to be 12 and the maple tree to be 180.

We have considered all of the possible cases. The age of the maple tree will be eleven times the age of the apple tree in 4 years when the apple tree is 14 and the maple tree is 154. Although it was not asked, today the apple tree is 10 and the maple tree is 150.

Solution 2:

Let a represent the age of the apple tree today and m represent the age of the maple tree today. Since the age of the maple tree is fifteen times the age of the apple tree, it follows that $m = 15a$.

Let n be the number of years until the age of the maple tree is eleven times the age of the apple tree, where n is a positive integer. In n years, the apple tree's age will be $a + n$ and the maple tree's age will be $m + n$. Then

$$
\begin{aligned}
m + n &= 11(a + n) \\
\text{But } m = 15a \text{ so} \qquad 15a + n &= 11(a + n) \\
15a + n &= 11a + 11n \\
4a &= 10n \\
2a &= 5n \\
\frac{2a}{5} &= n
\end{aligned}
$$

Since both a and n are positive integers, it follows that a is divisible by 5. But in the first solution, the possible ages for the apple tree were 7, 8, 9, 10, 11 and 12. The only multiple of 5 in this list is 10. Therefore $a = 10$ and then $n = 4$. Since $m = 15a$, it follows that $m = 15 \times 10 = 150$. Thus, today the apple tree is 10 and the maple tree is 150. In 4 years, the apple tree will be 14, the maple tree will be 154, and the maple tree's age will be eleven times the apple tree's age.