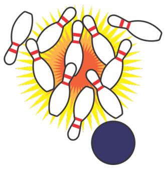

Problem of the Week
Problem D and Solution
Near Perfect

Problem

In ten-pin bowling, the highest possible score in a single game is 300. At one point in the bowling season, Fred F Stone had an average score of 177. In his next game he obtained a score of 199, which caused his average to increase to 178. After one more game Fred would like his average to be 183. Is it possible for Fred to accomplish this? If it is possible, what score does he need in his next game? If it is not possible, explain why not.

Solution

Solution 1

Let n be the number of games bowled to achieve his 177 average. His total points scored in n games is his average times n. Therefore, Fred has $177n$ total points in n games.

To compute his average after bowling the 199 game, we take his new total points and divide by $n+1$, the new number of games.

$$\begin{aligned}
\text{Average} &= \frac{\text{Total Points}}{\text{Games Played}} \\
178 &= \frac{177n+199}{n+1} \\
178(n+1) &= 177n+199 \\
178n+178 &= 177n+199 \\
n &= 21
\end{aligned}$$

Prior to bowling the 199 game, Fred had bowled 21 games. So after bowling the 199 game, Fred has bowled 22 games. Fred wants to have a 183 average after bowling his 23rd game. The difference between his total points after 23 games with a 183 average and his total points after bowling 22 games with a 178 average must be his score on the 23rd game.

$$\text{Score on 23rd Game} = 23 \times 183 - 22 \times 178 = 4209 - 3916 = 293$$

Therefore it is possible for Fred to raise his average from 178 to 183 in a single game, but he must bowl 293 in his next game to do this. This is a near perfect game.

Solution 2

Fred's score of 199 is $199 - 177 = 22$ points above his previous average. The game where Fred scored 199 raised his average 1 point. Therefore, his game with the 199 score must have been his 22nd game.

To raise his average 5 points in his 23rd game he must bowl $5 \times 23 = 115$ points above his 178 average. Therefore, he must bowl $178 + 115 = 293$ in his next game.

Therefore, it is possible for Fred to raise his average from 178 to 183 in a single game, but he must bowl 293 in his next game to do this. This is a near perfect game.

We can verify our results:

$$\begin{aligned}
&\text{Average on 21 games is 177.}\\
&\text{Average on 22 games} = \frac{21 \times 177 + 199}{22} = \frac{3916}{22} = 178\\
&\text{Average on 23 games} = \frac{22 \times 178 + 293}{23} = \frac{4209}{23} = 183
\end{aligned}$$