



Problem of the Week

Problem D and Solution

Count on This

Problem

Determine the number of integer values of n that satisfy the following inequality:

$$\frac{1}{9} \leq \frac{7}{n} \leq \frac{1}{5}$$

Solution

First notice that since $\frac{1}{9} \leq \frac{7}{n}$, and $\frac{1}{9}$ is positive, that means $\frac{7}{n}$ must be positive as well. It follows that n is positive.

Since $\frac{1}{9} = \frac{7}{63}$ and $\frac{1}{5} = \frac{7}{35}$, we can rewrite our inequality as follows:

$$\frac{7}{63} \leq \frac{7}{n} \leq \frac{7}{35}$$

Since the fractions are all positive and $n > 0$, this is true when $35 \leq n \leq 63$.

This is because if two fractions have the same numerator, then the larger fraction must have a smaller denominator, i.e. $\frac{2}{5} < \frac{2}{3}$.

Now we just need to count the number of values of n that satisfy $35 \leq n \leq 63$.

We could count them, but a faster way would be to do some simple math. Since n is an integer, there are $63 - 35 + 1 = 29$ possible values for n .