Problem of the Week
Problem C and Solution
In Their Prime

Problem

A *prime number* is an integer greater than 1 with exactly two different positive factors, 1 and the number itself. There are three children in a family. Each of their ages is a prime number. The sum of their ages is 41 and at least two of the children have ages that differ by 16. Determine all possibilities for the ages of the children.

Solution

We can start by listing all of the prime numbers less than 41. The possible prime ages are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37. We could actually eliminate some of the larger primes from this list since there are three different primes in the sum.

Now we will look for all prime pairs from this list that differ by 16. The pairs include 3 and 19, 7 and 23, and 13 and 29. We will look at each of these pairs and determine the third number so that the sum of the three ages is 41.

For the pair 3 and 19, the third age would be $41 - 3 - 19 = 19$, which is prime. The ages of the three children would be 3, 19, and 19. This is a possible solution.

For the pair 7 and 23, the third age would be $41 - 7 - 23 = 11$, which is prime. The ages of the three children would be 7, 11, and 23. This is another possible solution.

For the pair 13 and 29, the sum of these two ages is $13 + 29 = 42$. This sum is already over 41, so this is not a possible solution.

Therefore, there are two possibilities for the ages of the children. The children are either 7, 11 and 23 years old or 3, 19 and 19 years old.

For further thought: How would the problem change if the words "prime number" were removed from the problem and replaced with "positive integer"?