Problem of the Week
Problem C and Solution
Homestyle Lemon Drink

Problem

A restaurant owner was experimenting with the taste of her homestyle lemon drink. She started with 60 litres of water. She removed 15 litres of the water and replaced it with 15 litres of pure lemon juice. After thoroughly stirring the new mixture, she discovered that it was too "lemony". So she removed 10 litres of the new mixture and replaced it with 10 litres of water. She thoroughly stirred the mixture and concluded that the new mixture was just right. Determine the ratio of pure lemon juice to water in the final 60 litre mixture.

Solution

We need to determine the amount of pure lemon juice and the amount of water in the final mixture.

The owner starts with 60 litres of water and no lemon juice. After removing 15 litres of water and adding 15 litres of pure lemon juice, she has 15 litres of pure lemon juice and $60 - 15 = 45$ litres of water. So $\frac{15}{60} = \frac{1}{4}$ of the new mixture is pure lemon juice and $\frac{45}{60} = \frac{3}{4}$ of the new mixture is water.

She then removes 10 litres of the new mixture, $\frac{1}{4}$ of which is pure lemon juice and $\frac{3}{4}$ of which is water. So the owner removes $\frac{1}{4} \times 10 = \frac{10}{4} = \frac{5}{2}$ litres of pure lemon juice and $\frac{3}{4} \times 10 = \frac{30}{4} = \frac{15}{2}$ litres of water.

Before adding another 10 litres of water she has $15 - \frac{5}{2} = \frac{30}{2} - \frac{5}{2} = \frac{25}{2}$ litres of pure lemon juice and $45 - \frac{15}{2} = \frac{90}{2} - \frac{15}{2} = \frac{75}{2}$ litres of water.

After adding the final 10 litres of water, she has $10 + \frac{75}{2} = \frac{20}{2} + \frac{75}{2} = \frac{95}{2}$ litres of water.

The final ratio of pure lemon juice to water is

$$\frac{25}{2} : \frac{95}{2} = 25 : 95 = 5 : 19.$$

Therefore, the final ratio of pure lemon juice to water in the homestyle lemon drink is $5 : 19$.