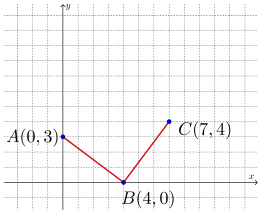



# Problem of the Week
## Problem C and Solution
## Penned In

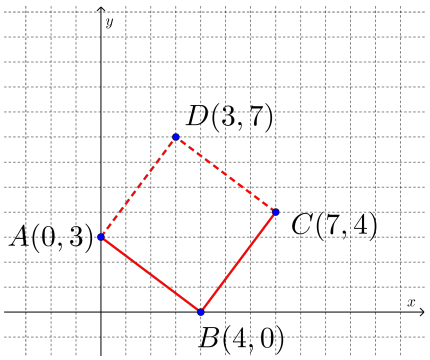
### Problem

A square enclosure, labelled $ABCD$, is sketched out on a piece of graph paper. Three of the vertices of the square $ABCD$ are located at $A(0, 3)$, $B(4, 0)$, and $C(7, 4)$. Determine the area of the square enclosure.

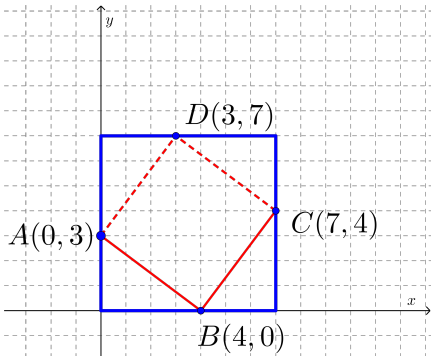
### Solution

### Solution 1

In this solution we will determine the area of $ABCD$ without using the Pythagorean Theorem.




We will first determine the coordinates of the fourth vertex $D$. To do so, observe that to get from $A$ to $B$, you would go down 3 units and right 4 units. To get from $B$ to $C$, you move 3 units to the right and then 4 units up. Continuing the pattern, going up 3 units and left 4 units, you get to $D(3, 7)$. Continuing, as a check, go left 3 units and down 4 units, and you arrive back at $A$. The coordinates of $D$ are $(3, 7)$.




Draw a box with horizontal and vertical sides so that each vertex of the square $ABCD$ is on one of the sides of the box. This creates a large square with sides of length 7 containing four congruent triangles and square $ABCD$. Each of the triangles has a base 4 units long and height 3 units long.

$$\begin{aligned}
\text{Area } ABCD &= \text{Area of Large Square} - 4 \times \text{Area of One Triangle} \\
&= \text{Length} \times \text{Width} - 4 \times (\text{Base} \times \text{Height} \div 2) \\
&= 7 \times 7 - 4 \times (4 \times 3 \div 2) \\
&= 49 - 4 \times 6 \\
&= 49 - 24 \\
&= 25
\end{aligned}$$

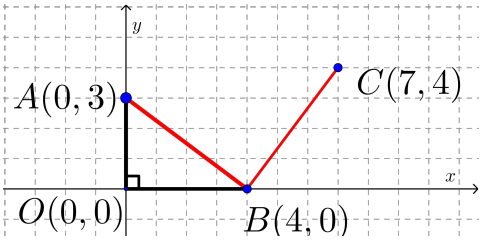
Therefore, the area of the enclosure is 25 units$^2$.

## Solution 2

In this solution we will determine the area of $ABCD$ using the Pythagorean Theorem.

Since $ABCD$ is a square, it is only necessary to find the length of one side. We can determine the area by squaring the length of the side.

Label the origin $O$.
$OA$, the distance from the origin to point $A$ on the $y$-axis, is 3 units. $OB$, the distance from the origin to point $B$ on the $x$-axis, is 4 units. Since $A$ lies on the $y$-axis and $B$ lies on the $x$-axis, $OAB$ forms a right-angled triangle.




Using the Pythagorean Theorem in right-angled $\triangle OAB$, we can find $AB^2$ which is $AB \times AB$, the area of the square.

$$\begin{aligned} AB^2 &= OA^2 + OB^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

Therefore, the area of the enclosure is 25 units$^2$.