Problem of the Week
Problem C and Solution
Around the Farm

Problem

Rahul has a farm he wishes to fence. The farm is the pentagon $ABCDE$, shown above. He knows that $ABCD$ is a 140 m by 150 m rectangle, as shown below. He also knows that E is 50 m from the side AB and 30 m from the side BC.
Determine the length of AE, the length of DE, and the perimeter of pentagon $ABCDE$.

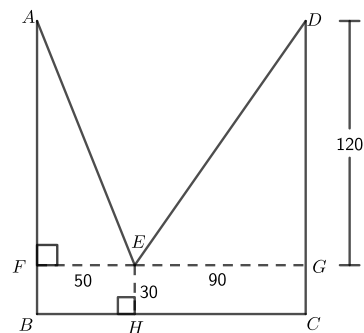## Solution

Let F be the point on AB with $EF = 50$ m.
Let H be the point on BC with $EH = 30$ m.
Extend EF to G on CD.
Since $ABCD$ is a rectangle and FG is perpendicular to AB,
then FG is perpendicular to CD and $FGCB$ is a rectangle.
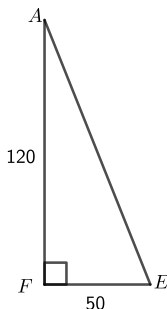Therefore, $FB = EH = GC = 30$ m.
Also, $DG = AF = AB - FB = 150 - 30 = 120$ m.

Since $\triangle AFE$ and $\triangle DGE$ are right-angled triangles, we can use the Pythagorean Theorem to determine the lengths of AE and DE.
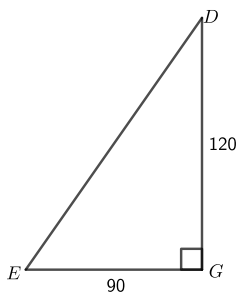
In $\triangle AFE$,

$$
\begin{aligned}
AE^2 &= AF^2 + FE^2 \\
&= 120^2 + 50^2 \\
&= 14\,400 + 2500 \\
&= 16\,900 \\
AE &= 130, \text{ since } AE > 0
\end{aligned}
$$

In $\triangle DGE$,

$$
\begin{aligned}
DE^2 &= DG^2 + EG^2 \\
&= 120^2 + 90^2 \\
&= 14\,400 + 8100 \\
&= 22\,500 \\
DE &= 150, \text{ since } DE > 0
\end{aligned}
$$

Therefore, $AE = 130$ m and $DE = 150$ m.
Also, the perimeter of pentagon $ABCDE$ is equal to
$$AB + BC + CD + DE + AE = 150 + 140 + 150 + 150 + 130 = 720 \text{ m.}$$