



Problem of the Week

Problem C and Solution

Order Up!

Problem

The letters w , x , y , and z each represent a different positive integer greater than 3. If we know that

$$\frac{1}{w-3} = \frac{1}{x+1} = \frac{1}{y+2} = \frac{1}{z-2}$$

then write w , x , y , and z in order from the letter that represents the smallest integer to the letter that represents the largest integer.

Solution

Solution 1:

Since the fractions are all equal and they all have a numerator of 1, that means that their denominators must all be equal. So $w - 3 = x + 1 = y + 2 = z - 2$.

Now let's suppose that $w = 10$. Then $w - 3 = 10 - 3 = 7$.

So $7 = x + 1 = y + 2 = z - 2$. We can make the following conclusions.

- Since $7 = x + 1$, that means $x = 7 - 1 = 6$.
- Since $7 = y + 2$, that means $y = 7 - 2 = 5$.
- Since $7 = z - 2$, that means $z = 7 + 2 = 9$.

So when $w = 10$, we have $x = 6$, $y = 5$, and $z = 9$. We can see that x is four less than w , y is five less than w , and z is one less than w . So when we write these in order from smallest to largest, we get y , x , z , w .

Solution 2:

As with Solution 1, we notice that since the fractions are all equal and they all have a numerator of 1, that means that their denominators must all be equal.

So $w - 3 = x + 1 = y + 2 = z - 2$. Let's add 3 to each expression.

$$\begin{array}{ccccccc} w - 3 & = & x + 1 & = & y + 2 & = & z - 2 \\ \downarrow +3 & & \downarrow +3 & & \downarrow +3 & & \downarrow +3 \\ w & = & x + 4 & = & y + 5 & = & z + 1 \end{array}$$

From this we can make the following conclusions.

- Since $w = z + 1$, that means w is 1 more than z , so $w > z$.
- Since $z + 1 = x + 4$, that means z is 3 more than x , so $z > x$.
- Since $x + 4 = y + 5$, that means x is 1 more than y , so $x > y$.

So when we write these in order from smallest to largest, we get y , x , z , w .