



Problem of the Week

Problem B and Solution

In an Orderly Fashion

Problem

- a) When we write the year 2021, we are writing two consecutive two-digit numbers (20 and 21). Find all the other years from 1000 to 2021 that are made up of two consecutive two-digit numbers written in order, and add them to the table below.
- b) Find the sum of the consecutive two-digit numbers for each year from part a), and add this to the table below. For example, for 2021, the sum is $20 + 21 = 41$. Describe the pattern formed by these sums.
- c) Find the product of the consecutive two-digit numbers for each of the first 5 years in the table. Then find the differences of these products, in order. For example, $10 \times 11 = 110$ and $11 \times 12 = 132$. The difference is $132 - 110 = 22$. You will have five products and four differences.
 - (i) What pattern is formed by the differences?
 - (ii) Use this pattern to find the remaining products, without multiplying.
- d) What sequence of numbers can you form by combining the numbers in the sum column and the difference column?

Year	Sum	Product	Difference
1011	$10 + 11 = 21$	$10 \times 11 = 110$	—
1112	$11 + 12 = 23$	$11 \times 12 = 132$	$132 - 110 = 22$
2021	$20 + 21 = 41$	$20 \times 21 = 420$	





Solution

- (a) There are 11 years from 1000 to 2021 that are made up of consecutive two-digit numbers written in order. They are
1011, 1112, 1213, 1314, 1415, 1516, 1617, 1718, 1819, 1920, and 2021
- (b) The table shows that each sum is 2 more than the previous sum.
- (c) (i) Each difference is 2 more than the previous difference.
If you fill in the difference column, then you can use that to fill in the product column. For example, the difference of products for 1112 is 22. This means that the next difference will be 24. Adding $132 + 24 = 156$, which is the next product. The entire table can be completed this way.
- (ii) The completed table is below.

Year	Sum	Product	Difference
1011	21	110	—
1112	23	132	22
1213	25	156	24
1314	27	182	26
1415	29	210	28
1516	31	240	30
1617	33	272	32
1718	35	306	34
1819	37	342	36
1920	39	380	38
2021	41	420	40

- (d) If we combine the numbers in these 2 columns, we get the sequence of whole numbers from 21 to 41.