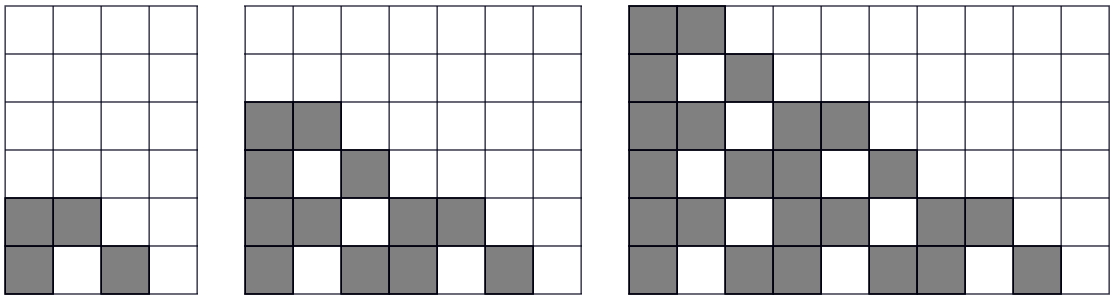
Problem of the Week
Problem B and Solution
Elsa's Pattern

Problem

Elsa has moved to the Kingdom of Summer and misses the beautiful frosty patterns of her icy homeland. She has decided to use a computer program and her printer to create some interesting snowflakes that she can keep all year long.

Below are the first three sizes of snowflakes she has created. She starts with the pattern in Stage 1. Stage 2 was created by adding 2 copies of the original pattern to the Stage 1 pattern. Stage 3 was created by adding 3 copies of the original pattern to the Stage 2 pattern. Stage 4 would be created by adding 4 copies of the original pattern to the Stage 3 pattern, and so on.



a) Using grid paper, draw Elsa's design for Stages 4 and 5. If you were to continue the pattern to further stages, would a stage ever have the same width and height?

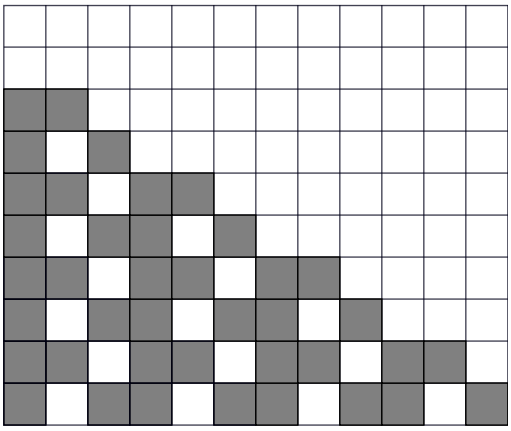
b) Complete the table below by entering how many grey and white squares Elsa has used at each stage, and calculating the ratio of grey to white squares. A white square must be within the pattern. That is, it must share a side with at least three grey squares.

Stage	Grey Squares	White Squares	Ratio of Grey to White Squares
1	4	1	4 : 1
2			
3			
4			
5			

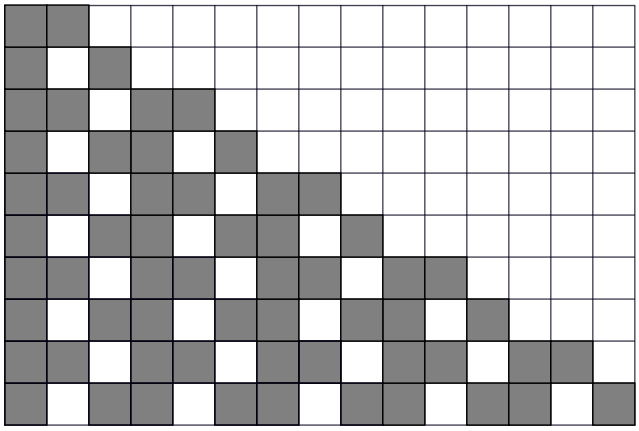
c) Using the table, how can you calculate the number of grey and white squares using only the stage number? Use your method to calculate the number of grey and white squares in Stage 6.

Solution

a) Elsa's designs for Stages 4 and 5 are shown.



Stage 4



Stage 5

Since the width increases by 3 squares and the height by 2 squares at each stage, the pattern will never have the same width and height.

b)

Stage	Grey Squares	White Squares	Ratio of Grey to White Squares
1	4	1	4 : 1
2	12	4	12 : 4 or 3 : 1
3	24	9	24 : 9 or 8 : 3
4	40	16	40 : 16 or 5 : 2
5	60	25	60 : 25 or 12 : 5

c) The number of white squares is equal to the stage number multiplied by itself. We call these numbers *perfect squares*.

$$\text{Number of white squares} = \text{Stage number} \times \text{Stage number}$$

To determine the number of grey squares in each stage, we add the stage number and the number of white squares, then multiply the result by 2. This is shown below using brackets, where we do the operations inside the brackets first.

$$\text{Number of grey squares} = (\text{Stage number} + \text{Number of white squares}) \times 2$$

So in Stage 6, the number of white squares is $6 \times 6 = 36$.

To determine the number of grey squares we first add $6 + 36 = 42$, then multiply by 2. So there are $42 \times 2 = 84$ grey squares in Stage 6.