Problem of the Week
Problem A and Solution
Coins vs. Dice

Problem

You are playing the newest board game Ticket to Clue. The goal of the game is to collect as many tokens as possible. When you land on a Chancey square you must pay 1 token for an opportunity to win 3 tokens. This Chancey square gives you two choices.

Coin Option	Dice Option
Flip two identical coins. Record which faces land up.	Roll two six-sided dice. Find their sum.
If the faces match, you WIN. If the faces don't match, you LOSE.	If the sum is 7 or greater, you WIN. If the sum is less than 7, you LOSE.

A) Experiment with each option. Try flipping coins and rolling dice at least 20 times each. Based on your experiments, which option would you choose?

B) Do you think the results of your experiments determined the best choice? Justify your answer.

C) If the Dice Option was changed so you were required to pay 15 tokens with a chance to win 17 tokens, would that change which option you would pick? Explain your choice.

Solution

A) Experimental results will vary. With a greater number of flips and rolls, it is more likely that the result will show that rolling the dice is a better option. However, it is possible that experimental results will be inconclusive or that they show that flipping the coin is a better option.

One way to get more reliable experimental results is to combine the outcomes from all students in the class. A larger sample size will produce better results.

B) We can logically show that the Dice Option gives a better chance of winning over the long term.

Let's first look at the Coin Option. When we flip two coins, there are four possible combinations of outcomes:

heads/heads, heads/tails, tails/heads, tails/tails

In half of these combinations the faces on the two coins match, so there is a 50-50 chance you will win with the Coin Option.

Now let's look at the Dice Option. Since there are 6 sides on each die, there are $6 \times 6 = 36$ possible combinations of the faces showing on the two dice. The table below shows the sums of all possible rolls.

Sums	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We see that a total of 21 combinations have a sum that is 7 or greater. That is more than half the possible outcomes, so the chance of winning with the Dice Option is better than 50-50. So in the long term, the Dice Option gives you a better chance of winning. Note that on a single turn, there is no guarantee that you would win with the Dice Option, so you are taking a chance either way.

C) In both cases, the net gain is 2 tokens when you win, and the probability of winning with the Dice Option is still slightly higher than the Coin Option. However, there are reasons to play the Coin Option in this case.

For example, in the real world the tokens in this problem would be a limited resource. Suppose you are only given 50 tokens to start the game. Imagine if you landed on the Chancey square three times in a row and lost each time.

If you chose the Dice Option each time, you paid a total of $3 \times 15 = 45$ tokens to play. If you chose the Coin Option each time, you paid a total of $3 \times 1 = 3$ tokens to play.

This means after three losses with the Dice Option, you would not have enough tokens to choose the Dice Option again. There is a substantially lower risk with the Coin Option.

Teacher's Notes

Statistically, we can answer part B) of this problem by calculating the *expected value* of the net gain for each option. When you WIN, you pay 1 token but win 3 tokens this is a *net gain* of +2. When you LOSE, you pay 1 token and do not win any tokens, so this is a net gain of −1.

The expected value is a weighted average based on the probability of each possible outcome. To find the expected value in this problem, we multiply the probability of winning by the net gain when we win, and add it to the result when we multiply the probability of losing by the net gain when we lose.

For the Coin Option, there is a 50% chance you WIN and 50% chance you LOSE. We can write 50% as a probability of $\frac{1}{2}$. Now we calculate the expected value as:

$$\left(\frac{1}{2} \times 2\right) + \left(\frac{1}{2} \times (-1)\right) = 1 + \left(\frac{-1}{2}\right) = \frac{1}{2}$$

For the Dice Option, there is a $\frac{21}{36}$ chance you WIN and $\frac{15}{36}$ chance you LOSE. Now we calculate the expected value as:

$$\left(\frac{21}{36} \times 2\right) + \left(\frac{15}{36} \times (-1)\right) = \frac{42}{36} + \left(\frac{-15}{36}\right) = \frac{42-15}{36} = \frac{27}{36} = \frac{3}{4}$$

Since $\frac{3}{4} > \frac{1}{2}$, then the expected net gain shows the Dice Option is a better choice.