



Problem of the Week

Problem E

Is It Valid?

Debit and credit cards contain account numbers which consist of many digits. When purchasing items online, you are often asked to type in your account number. Because there are so many digits, it is easy to type the number incorrectly. The last digit of the number is a specially generated check digit which can be used to quickly verify the validity of the number. A common algorithm used for verifying numbers is called the *Luhn Algorithm*. A series of operations are performed on the number and a final result is produced. If the final result ends in zero, the number is valid. Otherwise, the number is invalid.

The steps performed in the Luhn Algorithm are outlined in the flowchart below. Two examples are provided.

Example 1:

Number: 135792

Reversal: 297531

$$A = 2 + 7 + 3$$

$$= 12$$

$$2 \times 9 = 18$$

$$2 \times 5 = 10$$

$$2 \times 1 = 2$$

$$B = (1 + 8) + (1 + 0) + 2$$

$$= 9 + 1 + 2$$

$$= 12$$

$$C = 12 + 12 = 24$$

C does not end in zero.
The number is not valid.

Example 2:

Number: 1357987

Reversal: 7897531

$$A = 7 + 9 + 5 + 1$$

$$= 22$$

$$2 \times 8 = 16$$

$$2 \times 7 = 14$$

$$2 \times 3 = 6$$

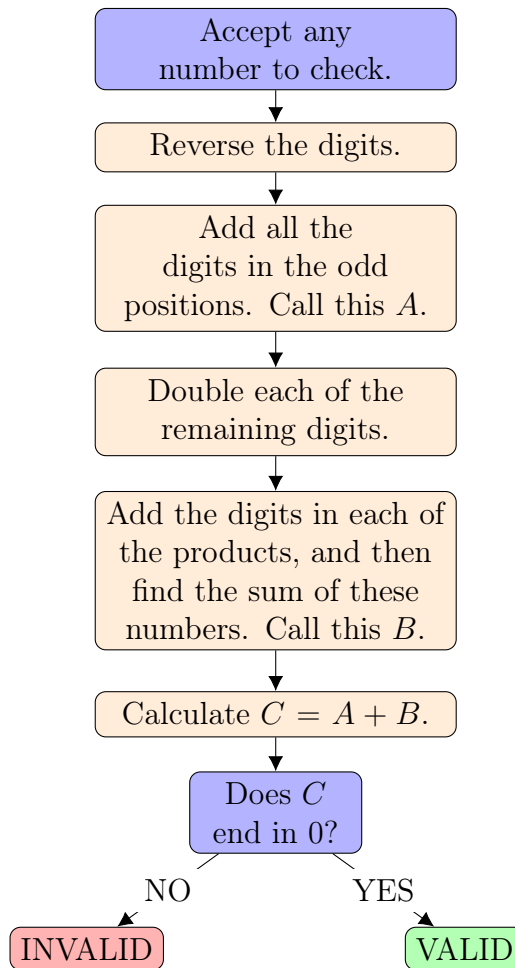
$$B = (1 + 6) + (1 + 4) + 6$$

$$= 7 + 5 + 6$$

$$= 18$$

$$C = 22 + 18 = 40$$

C ends in zero.
The number is valid.



Suppose the number 4633 $RT0R$ 481 is a valid number when verified by the Luhn Algorithm, where R and T are each integers from 0 to 9 such that $R \leq T$.

Determine all possible values of R and T .