Problem of the Week
Problem E and Solution
Angled III

Problem
In the circle with centre $R$ above, $PQ$ is a diameter. Point $S$ is a point on the circumference of the circle other than $P$ or $Q$. Determine the measure of $\angle PSQ$.

Solution
Join $S$ to the centre $R$. Since $RP$, $RQ$ and $RS$ are radii of the circle, $RP = RQ = RS$.

Since $RP = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x^\circ$.
Since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y^\circ$.

This new information is marked on the following diagram.

The angles in a triangle add to $180^\circ$, so in $\triangle PQS$
\[
\angle PSQ + \angle QPS + \angle PQS = 180^\circ
\]
\[
(x^\circ + y^\circ) + x^\circ + y^\circ = 180^\circ
\]
\[
2(x^\circ + y^\circ) = 180^\circ
\]
\[
x^\circ + y^\circ = 90^\circ
\]

But $\angle PSQ = x^\circ + y^\circ$, so $\angle PSQ = 90^\circ$.

This result is often expressed as a theorem for circles:
An angle($\angle PSQ$) inscribed in a circle by a diameter ($PQ$) of the circle is $90^\circ$. 