



Problem of the Week

Problem E and Solution

What are the Possibilities?

Problem

Determine all values of x that satisfy the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$.

Solution

Let's consider the ways that an expression of the form a^b can be 1:

- The base, a , is 1.

In this case, the exponent can be any value and we need to solve $x^2 - 5x + 5 = 1$.

$$\begin{aligned}x^2 - 5x + 5 &= 1 \\x^2 - 5x + 4 &= 0 \\(x - 4)(x - 1) &= 0\end{aligned}$$

So $x = 4$ or $x = 1$.

- The exponent, b , is 0.

In this case, the base can be any number other than 0 and we need to solve $x^2 + 4x - 60 = 0$.

$$\begin{aligned}x^2 + 4x - 60 &= 0 \\(x - 6)(x + 10) &= 0\end{aligned}$$

So $x = 6$ or $x = -10$.

When $x = 6$, the base is $6^2 - 5(6) + 5 = 11 \neq 0$. That is, when $x = 6$, the base does not equal 0.

When $x = -10$, the base is $(-10)^2 - 5(-10) + 5 = 155 \neq 0$. That is, when $x = -10$, the base does not equal 0.

- The base, a , is -1 and the exponent, b , is even.

We first need to solve $x^2 - 5x + 5 = -1$.

$$\begin{aligned}x^2 - 5x + 5 &= -1 \\x^2 - 5x + 6 &= 0 \\(x - 2)(x - 3) &= 0\end{aligned}$$

So $x = 2$ or $x = 3$.

When $x = 2$, the exponent is $2^2 + 4(2) - 60 = -48$, which is even.
Therefore, when $x = 2$, $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$.

When $x = 3$, the exponent is $3^2 + 4(3) - 60 = -39$, which is odd.
Therefore, when $x = 3$, $(x^2 - 5x + 5)^{x^2 + 4x - 60} = -1$. So $x = 3$ is not a solution.

Therefore, the values of x that satisfy $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ are $x = -10$, $x = 1$, $x = 2$, $x = 4$ and $x = 6$. There are five values of x which satisfy the equation.