Let $n$ be a positive integer. How many values of $n$ satisfy the following inequality?

$$(n - 1)(n - 3)(n - 5) \cdots (n - 2019)(n - 2021) \leq 0$$

**NOTE:** The product on the left side of the inequality consists of 1011 factors of the form $n - d$, where the value of $d$ starts at 1 and increases by 2 for each subsequent factor.