Problem of the Week
Problem E and Solution
What are the Possibilities?

Problem
Determine all values of $x$ that satisfy the equation $(x^2 - 5x + 5)x^{x^2+4x-60} = 1$.

Solution
Let’s consider the ways that an expression of the form $a^b$ can be 1:

• The base, $a$, is 1.
   In this case, the exponent can be any value and we need to solve $x^2 - 5x + 5 = 1$.
   
   $x^2 - 5x + 5 = 1$
   $x^2 - 5x + 4 = 0$
   $(x - 4)(x - 1) = 0$

   So $x = 4$ or $x = 1$.

• The exponent, $b$, is 0.
   In this case, the base can be any number other than 0 and we need to solve $x^2 + 4x - 60 = 0$.
   
   $x^2 + 4x - 60 = 0$
   $(x - 6)(x + 10) = 0$

   So $x = 6$ or $x = -10$.
   When $x = 6$, the base is $6^2 - 5(6) + 5 = 11 
eq 0$. That is, when $x = 6$, the base does not equal 0.
   When $x = -10$, the base is $(-10)^2 - 5(-10) + 5 = 155 
eq 0$. That is, when $x = -10$, the base does not equal 0.

• The base, $a$, is $-1$ and the exponent, $b$, is even.
   We first need to solve $x^2 - 5x + 5 = -1$.
   
   $x^2 - 5x + 5 = -1$
   $x^2 - 5x + 6 = 0$
   $(x - 2)(x - 3) = 0$

   So $x = 2$ or $x = 3$.
   When $x = 2$, the exponent is $2^2 + 4(2) - 60 = -48$, which is even.
   Therefore, when $x = 2$, $(x^2 - 5x + 5)x^{x^2+4x-60} = 1$.
   When $x = 3$, the exponent is $3^2 + 4(3) - 60 = -39$, which is odd.
   Therefore, when $x = 3$, $(x^2 - 5x + 5)x^{x^2+4x-60} = -1$. So $x = 3$ is not a solution.

Therefore, the values of $x$ that satisfy $(x^2 - 5x + 5)x^{x^2+4x-60} = 1$ are $x = -10, x = 1, x = 2, x = 4$ and $x = 6$. There are five values of $x$ which satisfy the equation.