



## Problem of the Week

### Problem D and Solution

#### Sibling Rivalry

#### Problem

Akira and Hideo are twins with different jobs. Akira earns five-eighths of what Hideo earns, but Akira's expenses are half of Hideo's. Akira ends up saving 40% of his income. What percentage of his income does Hideo save?

#### Solution

##### Solution 1: Using only one variable

Let  $h$  represent Hideo's income. Then Akira's income is  $\frac{5}{8}h$ .

Since Akira saves 40% of his income, his expenses are  $100\% - 40\% = 60\%$  of his income.

Therefore, Akira's expenses are  $60\% \times \frac{5}{8}h = \frac{60}{100} \times \frac{5}{8}h = \frac{3}{8}h$ .

Akira's expenses are one-half of Hideo's expenses so Hideo's expenses are twice Akira's expenses. Therefore, Hideo's expenses are  $2 \times \frac{3}{8}h = \frac{3}{4}h = 0.75h = 75\%$  of  $h$ . Since Hideo's expenses are 75% of his income, he saves  $100\% - 75\% = 25\%$  of his income.

Therefore, Hideo saves 25% of his income.

##### Solution 2: Using two variables

Let  $x$  represent Hideo's income and  $y$  represent Hideo's expenses.

Then Akira's income is  $\frac{5}{8}x$  and his expenses are  $\frac{1}{2}y$ .

Since Akira saves 40% of his income, his expenses are 60% of his income.

$$\begin{aligned}\frac{1}{2}y &= 0.60 \times \frac{5}{8}x \\ \frac{1}{2}y &= \frac{6}{10} \times \frac{5}{8}x \\ \frac{1}{2}y &= \frac{3}{8}x \\ y &= \frac{3}{4}x\end{aligned}$$

Hideo saves whatever is left of his income after expenses. Therefore Hideo saves

$$x - y = x - \frac{3}{4}x = \frac{1}{4}x = 0.25x = 25\% \text{ of } x.$$

Therefore, Hideo saves 25% of his income.

**Solution 3: Using two variables a bit differently**

Let  $8x$  represent Hideo's income and  $2y$  represent Hideo's expenses. Then Akira's income is  $\frac{5}{8}(8x) = 5x$  and his expenses are  $\frac{1}{2}(2y) = y$ .

Since Akira saves 40% of his income, his expenses are 60% of his income.

$$y = 0.60 \times 5x$$

$$y = \frac{6}{10} \times 5x$$

$$y = 3x$$

Hideo earns  $8x$  and his expenses are  $2y$  so his savings are  $8x - 2y$ . We want the ratio of his savings to his income,  $\frac{8x - 2y}{8x} = \frac{8x - 2(3x)}{8x} = \frac{2x}{8x} = \frac{1}{4}$  or 25% .

Therefore, Hideo saves 25% of his income.