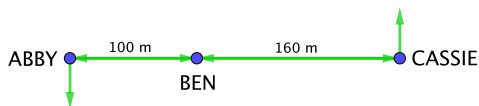




Problem of the Week

Problem D and Solution

Go Forth and Walk



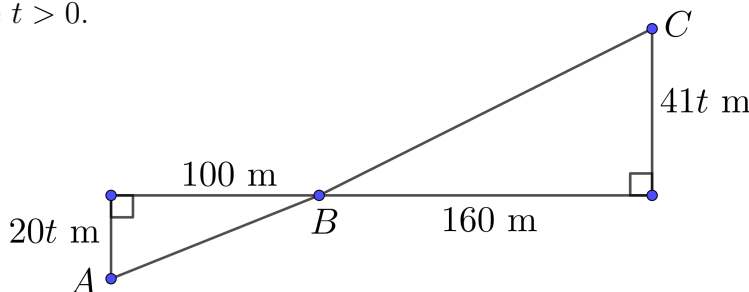
Problem

At noon three students, Abby, Ben, and Cassie, are standing so that Abby is 100 m west of Ben and Cassie is 160 m east of Ben. While Ben stays in his initial position, Abby begins walking south at a constant rate of $20 \frac{\text{m}}{\text{min}}$ and Cassie begins walking north at a constant rate of $41 \frac{\text{m}}{\text{min}}$. In how many minutes will the distance between Cassie and Ben be the twice the distance between Abby and Ben?

Solution

Solution 1

Let t represent the number of minutes until Cassie's distance to Ben is twice that of Abby's distance to Ben. In t minutes Abby will walk $20t$ m and Cassie will walk $41t$ m. The following diagram contains the information showing Abby's position, A , Ben's position, B , and Cassie's position, C , at time $t > 0$.



Since both triangles in the diagram are right-angled triangles, we can use the Pythagorean Theorem to set up an equation.

$$\begin{aligned}
 CB &= 2AB \\
 (CB)^2 &= (2AB)^2 \\
 (CB)^2 &= 4(AB)^2 \\
 (41t)^2 + (160)^2 &= 4[(20t)^2 + (100)^2] \\
 1681t^2 + 25600 &= 4[400t^2 + 10000] \\
 1681t^2 + 25600 &= 1600t^2 + 40000 \\
 81t^2 &= 14400 \\
 t^2 &= \frac{14400}{81} \\
 t &= \frac{120}{9}, \text{ since } t > 0 \\
 t &= \frac{40}{3} \text{ min}
 \end{aligned}$$

Therefore, in $13\frac{1}{3}$ minutes (13 minutes 20 seconds), Cassie's distance to Ben will be twice that of Abby's distance to Ben.

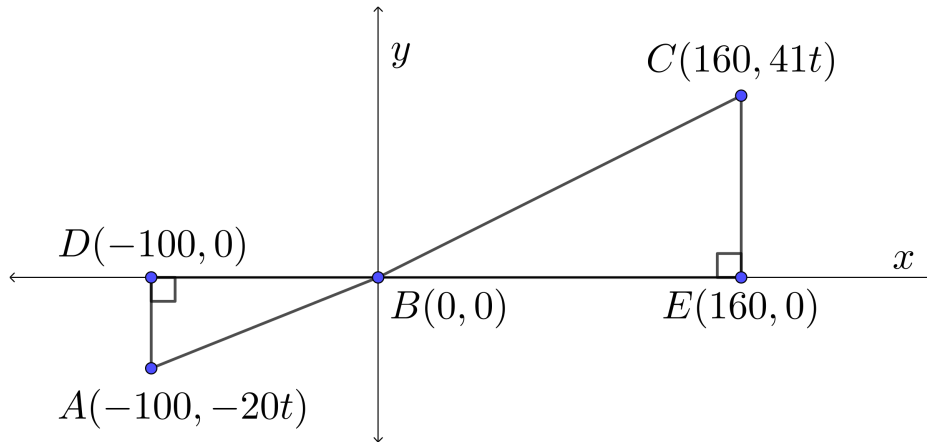
In Solution 2, an alternate solution that uses coordinate geometry is presented.

**Solution 2**

Represent Abby, Ben and Cassie's respective positions at noon as points on the x -axis so that Ben is positioned at the origin $B(0, 0)$, Abby is positioned 100 units left of Ben at $D(-100, 0)$ and Cassie is positioned 160 units right of Ben at $E(160, 0)$.

Let t represent the number of minutes until Cassie's distance to Ben is twice that of Abby's distance to Ben.

In t minutes Abby will walk south $20t$ m to the point $A(-100, -20t)$. In t minutes Cassie will walk north $41t$ m to the point $C(160, 41t)$.



The distance from a point $P(x, y)$ to the origin can be found using the formula $d = \sqrt{x^2 + y^2}$.

Then $AB = \sqrt{(-100)^2 + (-20t)^2} = \sqrt{10000 + 400t^2}$ and
 $CB = \sqrt{(160)^2 + (41t)^2} = \sqrt{25600 + 1681t^2}$.

$$\begin{aligned}
 CB &= 2AB \\
 \sqrt{25600 + 1681t^2} &= 2\sqrt{10000 + 400t^2} \\
 \text{Squaring both sides, } 25600 + 1681t^2 &= 4(10000 + 400t^2) \\
 25600 + 1681t^2 &= 40000 + 1600t^2 \\
 81t^2 &= 14400 \\
 t^2 &= \frac{14400}{81} \\
 t &= \frac{120}{9}, \text{ since } t > 0 \\
 t &= \frac{40}{3} \text{ min}
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Therefore, in $13\frac{1}{3}$ minutes (13 minutes 20 seconds), Cassie's distance to Ben will be twice that of Abby's distance to Ben.