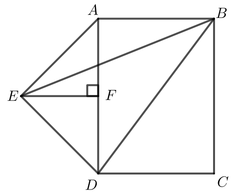


Problem of the Week

Problem D and Solution

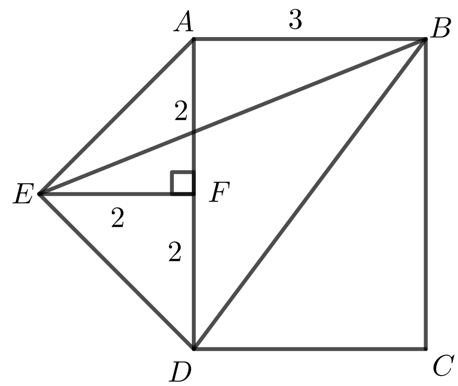
That Triangle

Problem

In the diagram, $ABCD$ is a rectangle. Point E is outside the rectangle so that $\triangle AED$ is an isosceles right-angled triangle with hypotenuse AD. Point F is the midpoint of AD, and EF is perpendicular to AD. If $BC = 4$ and $AB = 3$, determine the area of $\triangle EBD$.

Solution

Since $ABCD$ is a rectangle then $AD = BC = 4$. Since F is the midpoint of AD, then $AF = FD = 2$.



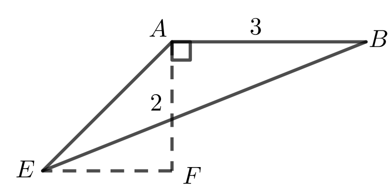
Since $\triangle AED$ is an isosceles right-angled triangle, then $\angle EAD = 45°$.
Now in $\triangle EAF$,
$\angle EAF = \angle EAD = 45°$ and $\angle AFE = 90°$.
Since the sum of the angles in a triangle is $180°$, then $\angle AEF = 180° - 90° - 45° = 45°$. Therefore, $\triangle EAF$ has two equal angles and is therefore an isosceles right-angled triangle.
Therefore, $EF = AF = 2$.

From this point we are going to look at two different solutions.

Solution 1:

We calculate the area of $\triangle EBD$ by adding the areas of $\triangle BAD$ and $\triangle AED$ and subtracting the area of $\triangle ABE$.
Since $AB = 3$, $DA = 4$, and $\angle DAB = 90°$, then the area of $\triangle BAD$ is $\frac{1}{2}(3)(4) = 6$.
Since $AD = 4$, $EF = 2$, and EF is perpendicular AD, then the area of $\triangle AED$ is $\frac{1}{2}(4)(2) = 4$.

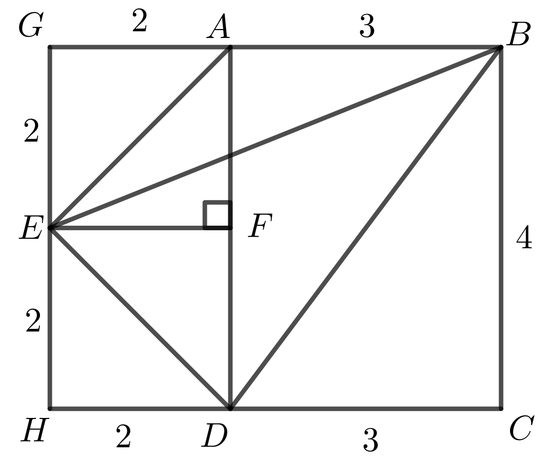
At the right, when we look at $\triangle ABE$ with the base being AB, then its height is the length of AF. Therefore, the area of $\triangle ABE$ is $\frac{1}{2}(3)(2) = 3$.



Therefore, the area of $\triangle EBD$ is $6 + 4 - 3 = 7$.

Solution 2:

Extend BA to G and CD to H so that GH is perpendicular to each GB and HC and so that GH passes through E.
Each of $GAFE$ and $EFDH$ has three right angles (at G, A, and F, and F, D, and H, respectively), so each of these is a rectangle.
Since $AF = EF = FD = 2$, then each of $GAFE$ and $EFDH$ is a square with side length 2.
Now $GBCH$ is a rectangle with $GB = 2 + 3 = 5$ and $BC = 4$.



The area of $\triangle EBD$ is equal to the area of rectangle $GBCH$ minus the areas of $\triangle EGB$, $\triangle BCD$, and $\triangle DHE$.

Rectangle $GBCH$ is 5 by 4, and so has area $5 \times 4 = 20$.
Since $EG = 2$ and $GB = 5$ and EG is perpendicular to GB,
then the area of $\triangle EGB$ is $\frac{1}{2}(EG)(GB) = \frac{1}{2}(2)(5) = 5$.
Since $BC = 4$ and $CD = 3$ and BC perpendicular to CD,
then the area of $\triangle BCD$ is $\frac{1}{2}(BC)(CD) = \frac{1}{2}(4)(3) = 6$.
Since $DH = HE = 2$ and DH is perpendicular to EH,
then the area of $\triangle DHE$ is $\frac{1}{2}(DH)(HE) = \frac{1}{2}(2)(2) = 2$.

Therefore, the area of $\triangle EBH$ is $20 - 5 - 6 - 2 = 7$.