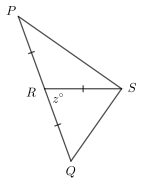


Problem of the Week
Problem D and Solution
Angled II

Problem

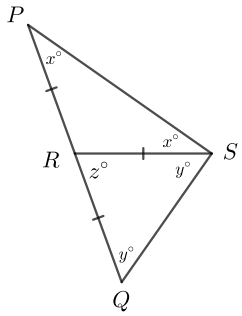
In $\triangle PQS$ above, R lies on PQ such that $PR = RQ = RS$ and $\angle QRS = z°$.
Determine the measure of $\angle PSQ$.

Solution

Solution 1

In $\triangle PRS$, since $PR = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x°$.

Similarly, in $\triangle QRS$, since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y°$.



Since PRQ is a straight line, $\angle PRS + \angle QRS = 180°$. Since $\angle QRS = z°$, we have $\angle PRS = 180 - z°$.

The angles in a triangle sum to 180°, so in $\triangle PRS$

$$\begin{aligned} \angle RPS + \angle RSP + \angle PRS &= 180° \\ x° + x° + 180 - z° &= 180° \\ 2x &= z \\ x &= \frac{z}{2} \end{aligned}$$

The angles in a triangle sum to 180°, so in $\triangle QRS$

$$\begin{aligned} \angle RQS + \angle RSQ + \angle QRS &= 180° \\ y° + y° + z° &= 180° \\ 2y &= 180 - z \\ y &= \frac{180 - z}{2} \end{aligned}$$

Then $\angle PSQ = \angle RSP + \angle RSQ = x° + y° = \frac{z}{2}° + \left(\frac{180-z}{2}\right)° = \left(\frac{180}{2}\right)° = 90°$.

Therefore, the measure of $\angle PSQ$ is 90°.

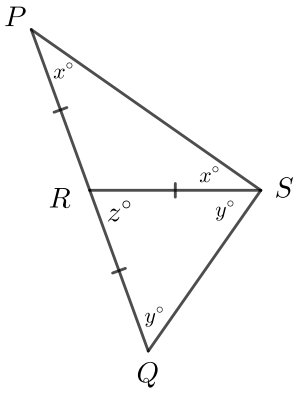
See Solution 2 for a more general approach to the solution of this problem.

It turns out that it is not necessary to determine expressions for x and y in terms of z to solve this problem.

Solution 2

In $\triangle PRS$, since $PR = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x°$.

Similarly, in $\triangle QRS$, since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y°$.



The angles in a triangle sum to 180°, so in $\triangle PQS$

$$\begin{aligned} \angle QPS + \angle PSQ + \angle PQS &= 180° \\ x° + (x° + y°) + y° &= 180° \\ (x° + y°) + (x° + y°) &= 180° \\ 2(x° + y°) &= 180° \\ x° + y° &= 90° \end{aligned}$$

But $\angle PSQ = \angle RSP + \angle RSQ = x° + y° = 90°$.

Therefore, the measure of $\angle PSQ$ is 90°.