Problem of the Week
Problem D and Solution
Order Four Cards

Problem

Four playing cards are placed in a row, from left to right. Each card is of a different suit. One card is a club (♣), one card is a diamond (♢), one card is a heart (♡), and one card is a spade (♠), not necessarily in that order. There is a different integer between 1 and 10, inclusive, on each card (an ace is read as a 1).

Using the following clues, determine the exact order of the cards, from left to right, including the suit and number.

1. The heart and the spade are immediately beside each other, in that order, from left to right.

2. The diamond is not the first (leftmost) card, and the club is not the fourth (rightmost) card.

3. The 7 is somewhere to the left of the club and the 3 is somewhere to the right of the club.

4. The two cards whose values are still unknown add to nine.

5. From left to right, the cards are arranged from largest to smallest.

Solution

Let C be the club, D be the diamond, H be the heart, and S be the spade.

Using the first clue, we can determine that there are only three possible placements for H and S, namely $(H, S, \square, \square)$, $(\square, H, S, \square)$ or $(\square, \square, H, S)$.

We will use the second clue to determine where the C and D must be placed in each possibility. For $(H, S, \square, \square)$, the order must be (H, S, C, D), since C cannot go last. For $(\square, H, S, \square)$, the order must be (C, H, S, D), since D cannot be first and C cannot be last. For $(\square, \square, H, S)$, the order must be (C, D, H, S), since D cannot go first.

The third clue says that 7 is somewhere to the left of the club. This means that the club cannot be furthest to the left. Therefore, we can rule out two of the three possibilities leaving (H, S, C, D) as the only possibility. Using the third clue further, we know that a 7 is somewhere to the left of the club and a 3 is somewhere to the right of the club. So the 7 is in the first or second spot, the 3 of diamonds is in the fourth spot. At this point we know the order is either $(7H, S, C, 3D)$ or $(H, 7S, C, 3D)$.

From the fourth clue we know that the remaining pair of numbers add to 9. Since numbers are used only once and the numbers 7 and 3 are already used, we can rule out the following pairs: 7 and 2, 6 and 3. That leaves only two possible pairs: 8 and 1, and 5 and 4. Using the fifth clue, we can place the numbers from largest to smallest and see if the solution is valid. Trying 8 and 1 in our possible solutions we obtain $(7H, 8S, 1C, 3D)$ or $(8H, 7S, 1C, 3D)$. Neither of these are valid since all the card values would not be arranged from largest to smallest. Trying 5 and 4 in our possible solutions we obtain $(7H, 5S, 4C, 3D)$ or $(5H, 7S, 4C, 3D)$. Only the first of these solutions satisfies the condition that all the card values are arranged from largest to smallest.

Therefore, the cards are placed in the following order from left to right: 7 of hearts $(7\heartsuit)$, then 5 of spades $(5\spadesuit)$, then 4 of clubs $(4\clubsuit)$, and then 3 of diamonds $(3\diamondsuit)$.