Problem of the Week
Problem D and Solution
Count on This

Problem
Determine the number of integer values of \( n \) that satisfy the following inequality:

\[
\frac{1}{9} \leq \frac{7}{n} \leq \frac{1}{5}
\]

Solution
First notice that since \( \frac{1}{9} \leq \frac{7}{n} \), and \( \frac{1}{9} \) is positive, that means \( \frac{7}{n} \) must be positive as well. It follows that \( n \) is positive.

Since \( \frac{1}{9} = \frac{7}{63} \) and \( \frac{1}{5} = \frac{7}{35} \), we can rewrite our inequality as follows:

\[
\frac{7}{63} \leq \frac{7}{n} \leq \frac{7}{35}
\]

Since the fractions are all positive and \( n > 0 \), this is true when \( 35 \leq n \leq 63 \).

This is because if two fractions have the same numerator, then the larger fraction must have a smaller denominator, i.e. \( \frac{2}{5} < \frac{2}{3} \).

Now we just need to count the number of values of \( n \) that satisfy \( 35 \leq n \leq 63 \).

We could count them, but a faster way would be to do some simple math. Since \( n \) is an integer, there are \( 63 - 35 + 1 = 29 \) possible values for \( n \).