



## Problem of the Week

### Problem C and Solution

#### Shape On



#### Problem

A necklace is to be created that contains only square shapes, circular shapes, and triangular shapes. A total of 180 of these shapes will be strung on the necklace in the following sequence: 1 square, 1 circle, 1 triangle, 2 squares, 2 circles, 2 triangles, 3 squares, 3 circles, 3 triangles, with the number of each shape type increasing by one every time a new group of shapes is placed. The diagram illustrates how the first 18 shapes would be strung.

Once the necklace is completed, how many of each shape would the necklace contain?

#### Solution

An equal number of square shapes, circular shapes and triangular shapes occur after

$$3(1) = 3 \text{ shapes are placed,}$$

$$3(1) + 3(2) = 3 + 6 = 9 \text{ shapes are placed,}$$

$$3(1) + 3(2) + 3(3) = 3 + 6 + 9 = 18 \text{ shapes are placed, and so on.}$$

The greatest total that can be placed with equal numbers of squares, circles and triangles is

$$\begin{aligned} 3(1) + 3(2) + 3(3) + 3(4) + 3(5) + 3(6) + 3(7) + 3(8) + 3(9) + 3(10) \\ = 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 \\ = 165 \end{aligned}$$

At this point there are  $165 \div 3 = 55$  of each of the three shapes. We are at a point the last 30 shapes placed are 10 squares, 10 circles and 10 triangles, in that order.

The next group would have 11 of each shape if all of the shapes could be placed. However, there are only  $180 - 165 = 15$  shapes left to place. We are able to place 11 square shapes leaving 4 shapes left to place. At this point there are  $55 + 11 = 66$  square shapes on the necklace. The final 4 shapes would be circular. And at this point there would be 59 circular shapes on the necklace. No more triangular shapes can be added so the total number of triangular shapes remains at 55.

Therefore, the completed necklace will contain 66 square shapes, 59 circular shapes, and 55 triangular shapes.