Problem

Rahul has a farm he wishes to fence. The farm is the pentagon $ABCDE$, shown above. He knows that $ABCD$ is a 140 m by 150 m rectangle, as shown below. He also knows that $E$ is 50 m from the side $AB$ and 30 m from the side $BC$.

Determine the length of $AE$, the length of $DE$, and the perimeter of pentagon $ABCDE$.

Solution

Let $F$ be the point on $AB$ with $EF = 50$ m.
Let $H$ be the point on $BC$ with $EH = 30$ m.
Extend $EF$ to $G$ on $CD$.
Since $ABCD$ is a rectangle and $FG$ is perpendicular to $AB$, then $FG$ is perpendicular to $CD$ and $FGCB$ is a rectangle. Therefore, $FB = EH = GC = 30$ m.
Also, $DG = AF = AB - FB = 150 - 30 = 120$ m.

Since $\triangle AFE$ and $\triangle DGE$ are right-angled triangles, we can use the Pythagorean Theorem to determine the lengths of $AE$ and $DE$.

In $\triangle AFE$,

\[
AE^2 = AF^2 + FE^2 \\
= 120^2 + 50^2 \\
= 14400 + 2500 \\
= 16900 \\
AE = 130, \text{ since } AE > 0
\]

In $\triangle DGE$,

\[
DE^2 = DG^2 + EG^2 \\
= 120^2 + 90^2 \\
= 14400 + 8100 \\
= 22500 \\
DE = 150, \text{ since } DE > 0
\]

Therefore, $AE = 130$ m and $DE = 150$ m.
Also, the perimeter of pentagon $ABCDE$ is equal to

\[
AB + BC + CD + DE + AE = 150 + 140 + 150 + 150 + 130 = 720 \text{ m}.
\]