

Problem of the Week

Problem C and Solution

Around the Farm

Problem

Rahul has a farm he wishes to fence. The farm is the pentagon $ABCDE$, shown above. He knows that $ABCD$ is a 140 m by 150 m rectangle, as shown below. He also knows that E is 50 m from the side AB and 30 m from the side BC .

Determine the length of AE , the length of DE , and the perimeter of pentagon $ABCDE$.

Solution

Let F be the point on AB with $EF = 50$ m.

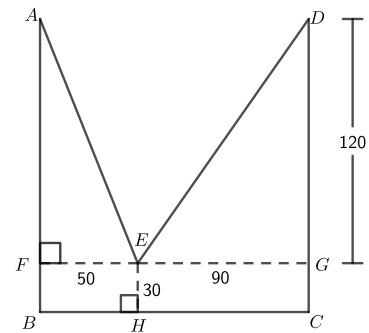
Let H be the point on BC with $EH = 30$ m.

Extend EF to G on CD .

Since $ABCD$ is a rectangle and FG is perpendicular to AB , then FG is perpendicular to CD and $FGCB$ is a rectangle.

Therefore, $FB = EH = GC = 30$ m.

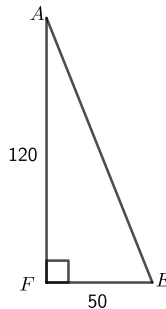
Also, $DG = AF = AB - FB = 150 - 30 = 120$ m.



Since $\triangle AFE$ and $\triangle DGE$ are right-angled triangles, we can use the Pythagorean Theorem to determine the lengths of AE and DE .

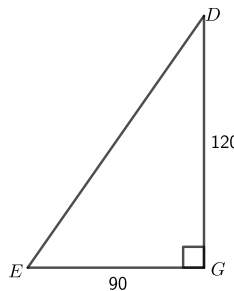
In $\triangle AFE$,

$$\begin{aligned} AE^2 &= AF^2 + FE^2 \\ &= 120^2 + 50^2 \\ &= 14\,400 + 2\,500 \\ &= 16\,900 \\ AE &= 130, \text{ since } AE > 0 \end{aligned}$$



In $\triangle DGE$,

$$\begin{aligned} DE^2 &= DG^2 + EG^2 \\ &= 120^2 + 90^2 \\ &= 14\,400 + 8\,100 \\ &= 22\,500 \\ DE &= 150, \text{ since } DE > 0 \end{aligned}$$



Therefore, $AE = 130$ m and $DE = 150$ m.

Also, the perimeter of pentagon $ABCDE$ is equal to

$$AB + BC + CD + DE + AE = 150 + 140 + 150 + 150 + 130 = 720 \text{ m.}$$