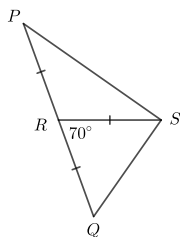




## Problem of the Week

### Problem C and Solution

### Angled



#### Problem

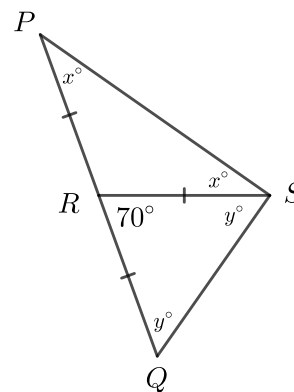
In  $\triangle PQS$  above,  $R$  lies on  $PQ$  such that  $PR = RQ = RS$  and  $\angle QRS = 70^\circ$ . Determine the measure of  $\angle PSQ$ .

#### Solution

##### Solution 1

In  $\triangle PRS$ , since  $PR = RS$ ,  $\triangle PRS$  is isosceles and  $\angle RPS = \angle RSP = x^\circ$ .

Similarly, in  $\triangle QRS$ , since  $RQ = RS$ ,  $\triangle QRS$  is isosceles and  $\angle RQS = \angle RSQ = y^\circ$ .



Since  $PRQ$  is a straight line,  $\angle PRS + \angle QRS = 180^\circ$ . Since  $\angle QRS = 70^\circ$ , we have  $\angle PRS = 110^\circ$ .

The angles in a triangle sum to  $180^\circ$ , so in  $\triangle PRS$

$$\begin{aligned}\angle RPS + \angle RSP + \angle PRS &= 180^\circ \\ x^\circ + x^\circ + 110^\circ &= 180^\circ \\ 2x &= 70 \\ x &= 35\end{aligned}$$

The angles in a triangle sum to  $180^\circ$ , so in  $\triangle QRS$

$$\begin{aligned}\angle RQS + \angle RSQ + \angle QRS &= 180^\circ \\ y^\circ + y^\circ + 70^\circ &= 180^\circ \\ 2y &= 110 \\ y &= 55\end{aligned}$$

Then  $\angle PSQ = \angle RSP + \angle RSQ = x^\circ + y^\circ = 35^\circ + 55^\circ = 90^\circ$ .

Therefore, the measure of  $\angle PSQ$  is  $90^\circ$ .

See Solution 2 for a more general approach to the solution of this problem.

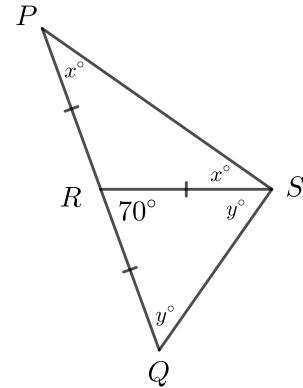


It turns out that it is not necessary to determine the values of  $x$  and  $y$  to solve this problem.

### Solution 2

In  $\triangle PRS$ , since  $PR = RS$ ,  $\triangle PRS$  is isosceles and  $\angle RPS = \angle RSP = x^\circ$ .

Similarly, in  $\triangle QRS$ , since  $RQ = RS$ ,  $\triangle QRS$  is isosceles and  $\angle RQS = \angle RSQ = y^\circ$ .



The angles in a triangle sum to  $180^\circ$ , so in  $\triangle PQS$

$$\angle QPS + \angle PSQ + \angle PQS = 180^\circ$$

$$x^\circ + (x^\circ + y^\circ) + y^\circ = 180^\circ$$

$$(x^\circ + y^\circ) + (x^\circ + y^\circ) = 180^\circ$$

$$2(x^\circ + y^\circ) = 180^\circ$$

$$x^\circ + y^\circ = 90^\circ$$

But  $\angle PSQ = \angle RSP + \angle RSQ = x^\circ + y^\circ = 90^\circ$ .

Therefore, the measure of  $\angle PSQ$  is  $90^\circ$ .