Problem of the Week
Problem C and Solution
CADET

Problem

We can take any word and rearrange all the letters to get another "word". These new "words" may be nonsensical. For example, you can rearrange the letters in $MATH$ to get $MTHA$. Geordie wants to rearrange all the letters in the word $CADET$. However, he uses the following rules:

- the letters A and D must be beside each other, and
- the letters E and T must be beside each other.

How many different arrangements of the word $CADET$ can Geordie make if he follows these rules?

Solution

We will look at a systematic way of counting the arrangements by first looking at a simpler example.
Let's start with a three letter word. How many different ways can we arrange the letters of the word SPY?
If we list all the arrangements, we get 6 arrangements.
They are: $SPY, SYP, PYS, PSY, YPS, YSP$.

There is another way to count these 6 cases without listing them all out:
If we consider the first letter, there are 3 possibilities. For each of these possibilities, there are 2 remaining options for the second letter. Finally, once the first and second letters are set, there is only one possibility left for the last letter.
To get the number of possible arrangements, we multiply $\underline{\ 3\ } \times \underline{\ 2\ } \times \underline{\ 1\ } = 6$.

Let's look at our problem now.
If we consider A and D as the single "letter" AD, and E and T as the single "letter" ET, we now have only the three "letters" C, AD, and ET.
As we saw above, there are $\underline{\ 3\ } \times \underline{\ 2\ } \times \underline{\ 1\ } = 6$ ways to arrange the three "letters".
These arrangements are:
$CADET, CETAD, ADCET, ADETC, ETCAD, ETADC$.
(We will refer to these as the original six.)

However, note that the question says A and D must be beside each other. This means they could appear as AD or DA. Similarly, E and T could appear as ET or TE.

Let's take a look at the first word, $CADET$.
We could switch AD to DA. This means $CADET$ becomes $CDAET$, which is a valid arrangement.
We could also switch ET to TE. This means $CADET$ becomes $CADTE$, which is a valid arrangement.
We could also switch both AD to DA and ET to TE. This means $CADET$ becomes $CDATE$, which is a valid arrangement.

We can do these changes for each of the original six. We list these arrangements in the table below.

One of the Original Six	Switch AD	Switch ET	Switch both AD and ET
$CADET$	$CDAET$	$CADTE$	$CDATE$
$CETAD$	$CETDA$	$CTEAD$	$CTEDA$
$ADCET$	$DACET$	$ADCTE$	$DACTE$
$ADETC$	$DAETC$	$ADTEC$	$DATEC$
$ETCAD$	$ETCDA$	$TECAD$	$TECDA$
$ETADC$	$ETDAC$	$TEADC$	$TEDAC$

Therefore, there are 24 possible arrangements that Geordie can make when following the given rules.

NOTE: There is another way to count the number of arrangements. There are 6 ways to arrange the 3 "letters". There are 2 ways to arrange AD and there are 2 ways to arrange ET. To determine the total number of arrangements, we multiply $6 \times 2 \times 2$. This gives us 24 arrangements.