



Problem of the Week

Problem A and Solution

Voting for Books

Problem

Students at Norwood Public School are assigned to read one book each month. Each student in the school votes for one book type. The results are combined to decide what type of book will be assigned for next month's reading for the whole school. Each grade in the school has 24 students. Here are the results of the vote:

- Half of the Grade 3 students chose Science Fiction.
- One-quarter of the Grade 3 students chose Biography.
- One-quarter of the Grade 3 students chose Mystery.
- The students' votes in Grade 4 are equally divided between Science Fiction, Biography, and Mystery.
- One-third of the Grade 5 students chose Science Fiction and the rest of the class chose Mystery.

- A) If the votes in Grades 3, 4, and 5 are the only ones that are counted, which type of book should be read next month? Justify your answer.
- B) If half of the students in Grade 6 voted for Mystery and half voted for Biography, which type of book should be read next month based on the votes from Grades 3, 4, 5, and 6? Justify your answer.
- C) Suppose you know that Grades 3, 4, and 5 each have the same number of students, but this number is unknown. Given the same voting results described in the question, can you still answer the question in part A? Justify your answer.



Solution

A) We can look at each grade separately.

Grade 3

- Half of the Grade 3 students chose Science Fiction. Since $2 \times 12 = 24$, half of 24 is 12. So we know 12 students chose Science Fiction.
- One-quarter of the Grade 3 students chose Biography. Since $4 \times 6 = 24$, one-quarter of 24 is 6. So 6 students chose Biography.
- One-quarter of the Grade 3 students chose Mystery. Since one-quarter of 24 is 6, that means 6 students chose Mystery.

Grade 4

- The students' votes in Grade 4 were equally divided between the 3 book types. Since $3 \times 8 = 24$, we know 8 students chose Science Fiction, 8 students chose Biography, and 8 students chose Mystery.

Grade 5

- One-third of the Grade 5 students chose Science Fiction. Since $3 \times 8 = 24$, one-third of 24 is 8. So 8 students chose Science Fiction.
- The remaining students chose Mystery, so $24 - 8 = 16$ students chose Mystery.
- This also tells us that 0 students chose Biography.

When we total each book type, we see the following results:

$$\text{Science Fiction: } 12 + 8 + 8 = 28$$

$$\text{Biography: } 6 + 8 + 0 = 14$$

$$\text{Mystery: } 6 + 8 + 16 = 30$$

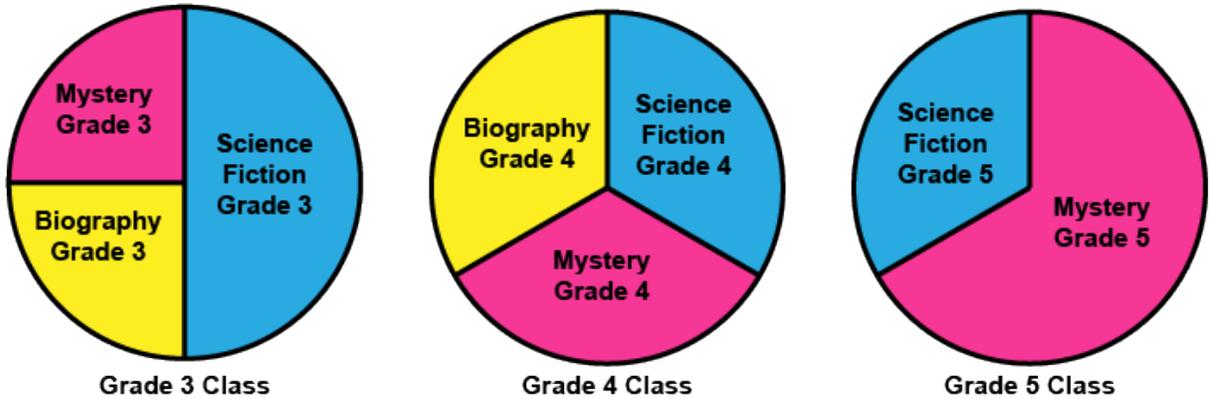
Based on the votes from these grades, next month's book will be a Mystery.

B) If we count the votes from Grade 6, we could do the same work that we did for part A of this question to determine the most popular type of book. However, we could also figure out the result without doing much extra work.

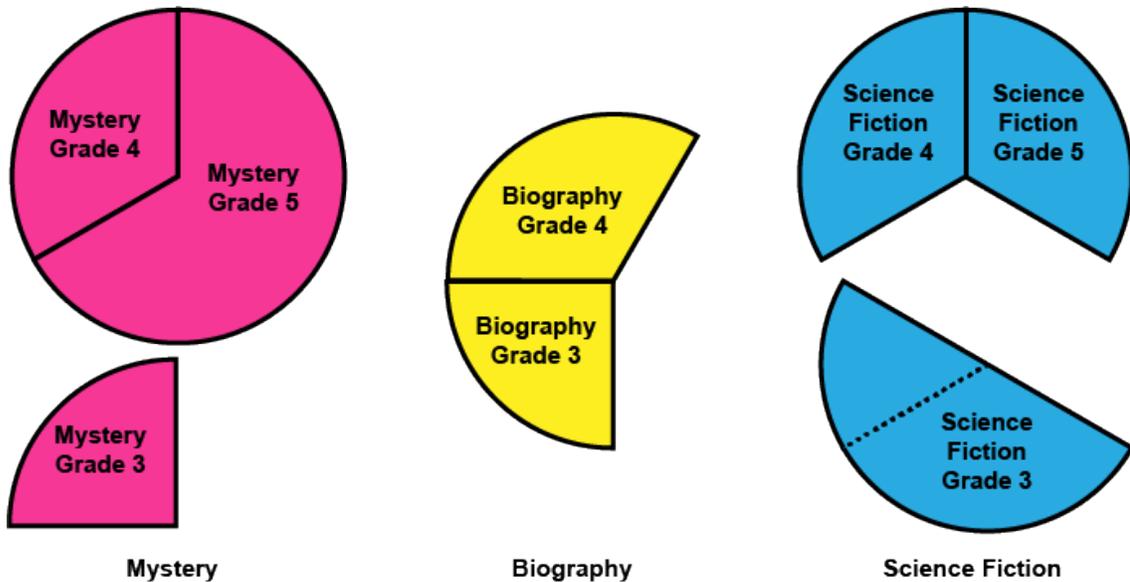
From part A, we know that there were more votes cast for Mystery than the other two choices. There were no votes in Grade 6 for Science Fiction, so its total will be unchanged. Since an equal number of students from Grade 6 voted for Mystery as voted for Biography, then we know that in total there will still be more votes for Mystery than either of the other types of books. So based on the votes in Grades 3, 4, 5, and 6, next month's book will be a Mystery.



C) If we do not know the number of students in each grade, we can still determine the winner of the vote. We could use diagrams where circles represent the whole grade and slices represent the fractions that voted for each type of book.



Then we could rearrange the pieces into groups for each type of book.



From this we can quickly see that Biography was the least popular type of book. It is a little less clear which of Mystery and Science Fiction is more popular. We could cut the "Science Fiction Grade 3" piece into two smaller pieces. One piece would be big enough to fill in the rest of the circle currently containing the fractions of Grade 4 and 5 students who voted for Science Fiction, and the other piece would be whatever is left over. Then we can compare the left over piece with the piece representing the students in Grade 3 who voted for Mystery. When we compare those two pieces, we see that the Mystery piece is larger. This means that the most popular type is Mystery.



Teacher's Notes

In Mathematics, when we come to conclusions we like to be sure that they are correct. We can use examples to quickly check if a conclusion is reasonable, but normally to prove something is correct we need to be generic.

For example, the *Pythagorean Theorem* states that in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. We could draw various right-angled triangles, and measure the lengths of their sides to see if the theorem holds. Since our drawing and measurements are imprecise, we will likely see some small differences in the resulting calculations. We would find that the numbers are pretty close to matching the theorem's statement and think that Pythagoras was probably right.

However, if we want to prove the theorem is correct, then we cannot use any particular triangle or lengths. We need to use mathematical techniques that describe the situation more generally.

However, in some cases we can make some specific assumptions when writing a justification or mathematical proof. The phrase, "*without loss of generality*" is used to identify a situation where we can make a specific assumption before starting a proof or justification. Stating that assumption often makes the proof simpler and easier to understand. However, it is important that the assumption does not affect the legitimacy of the logic in the proof, so we need to be careful.

In this problem, since it refers to fractions of a whole (i.e. the number of students in a class), and the classes are all the same size, we can determine the winner of the vote whether or not we know exactly how many students are in each class. However, it is easier to work with fractions of a specific number compared to fractions of an unknown variable. Suppose we doubled the size of the classes. The total number of votes in each case would also double, so this would not affect which genre received **more** votes. Had we been asked the question in part (C) without being told the number of students in each class in the original question, we could have started by choosing a particular size for each class. To make our calculations easier, we should pick a number that is easily divided into halves, quarters, and thirds. We do need to be careful though. Without knowing the specific size of the class we can know which genre won the vote, but we cannot know the specific margin of victory. Use this tool with caution.