

## Problem of the Week

### Problem E and Solution

### Spiral

#### Problem

A spiral of numbers is created, as shown above, starting with 1. If the pattern of the spiral continues, how will the numbers 2020, 2021, and 2022 appear in the spiral? (Will they appear left to right in a row? Right to left in a row? Down in a column? Up in a column? Down and then left? Up and then right?)

#### Solution

We are looking for a pattern which can be used to predict the positions of the numbers 2020, 2021, and 2022.

Observe a first “square” in the middle containing the numbers 1, 2, 3, and 4, with 4 in the lower right corner.

$$\begin{array}{cc}
 2 \rightarrow 3 & \\
 \uparrow & \downarrow \\
 1 & 4
 \end{array}$$

Starting with the 1, to get this “square” we need to add one number above and one number to right. We also need to add a number that is in the diagonal up and to the right. That is, we need to add three numbers to get the first “square”.

Extend the diagram to create a second “square”:

$$\begin{array}{ccc}
 9 & 2 \rightarrow 3 & \\
 \uparrow & \uparrow & \downarrow \\
 8 & 1 & 4 \\
 \uparrow & & \downarrow \\
 7 \leftarrow 6 \leftarrow 5 & & 
 \end{array}$$

To get this second “square”, we need to add two numbers below the first “square” and two numbers to the left of the first “square”. We also need to add a number that is in the diagonal down and to the left. That is, we need to add five numbers to get from the first “square” to the second “square”.

Extend the diagram to create a third “square”:

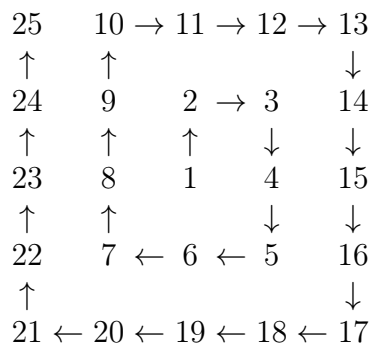
$$\begin{array}{cccc}
 10 \rightarrow 11 \rightarrow 12 \rightarrow 13 & & & \\
 \uparrow & & & \downarrow \\
 9 & 2 \rightarrow 3 & 14 & \\
 \uparrow & \uparrow & \downarrow & \downarrow \\
 8 & 1 & 4 & 15 \\
 \uparrow & & \downarrow & \downarrow \\
 7 \leftarrow 6 \leftarrow 5 & & 16 & 
 \end{array}$$





To get this third “square”, we need to add three numbers above the second “square” and three numbers to the right of the second “square”. We also need to add a number that is in the diagonal up and to the right. That is, we need to add seven numbers to get from the second “square” to the third “square”.

Extend the diagram to create a fourth “square”:



To get this fourth “square” we need to add four numbers below the third “square” and four numbers to the left of the third “square”. We also need to add a number that is in the diagonal down and to the left. That is, we need to add nine numbers to get from the third “square” to the fourth “square”.

If this pattern continues, the fifth “square” would ‘end’ with  $25 + 11 = 36$  in the bottom right position. The sixth “square” would end with  $36 + 13 = 49$  in the top left. Notice that the numbers 4, 9, 16, 25, 36, and 49 are all perfect squares. Also notice that the even perfect squares are in the bottom right corners and the odd perfect squares are in the top left corners. Now, 2020 lies between  $44^2 = 1936$  and  $45^2 = 2025$ . Since 2025 is an odd perfect square, 2025 will be in the top left corner of a “square”, and the left side of the “square” will look like this:



Therefore, 2020, 2021 and 2022 will be going up in a column.

