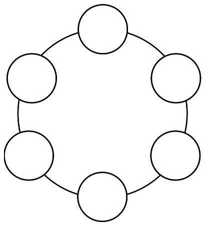


Problem of the Week

Problem D and Solution

A Circle of Numbers

Problem

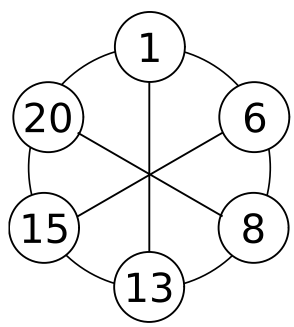
The numbers 1, 6, 8, 13, 15, and 20 can be placed in the circle above, each exactly once, so that the sum of each pair of numbers adjacent in the circle is a multiple of seven. In fact, there is more than one way to arrange the numbers in such a way in the circle. Determine all different arrangements. Note that we will consider two arrangements to be the same if one can be obtained from the other by a series of reflections and rotations.

Solution

We will start by writing down all the pairs of numbers that add to a multiple of 7.

Sum of 7	Sum of 14	Sum of 21	Sum of 28	Sum of 35
1,6	1,13	6,15	13,15	15,20
	6,8	8,13	8,20	
		1,20		

To show these connections visually, we can write the numbers in a circle and draw a line connecting numbers that add to a multiple of 7.

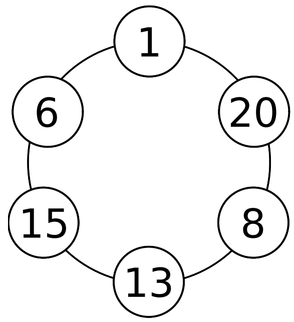


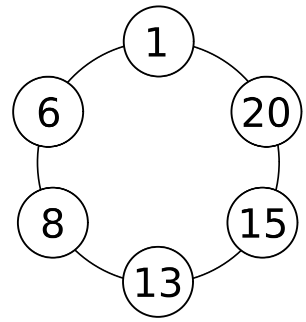
We will now determine all the different arrangements by looking at various cases. Note that in order for two arrangements to be different, at least some of the numbers need to be adjacent to different numbers.

Now, consider the possibilities for the numbers adjacent to 1. Since 6, 13, and 20 are the only numbers in our list that add with 1 to make a multiple of 7, there are three possible cases: 1 adjacent to 6 and 20, 1 adjacent to 6 and 13, and 1 adjacent to 13 and 20. We consider each case separately.

Case 1: 1 is adjacent to 6 and 20

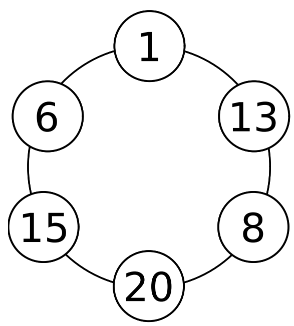
In this case, we can see from our table that 13 must be adjacent to 15 and 8, since 1 is no longer available. The two different ways to write such a circle are shown below.

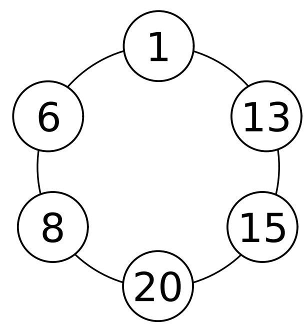




Case 2: 1 is adjacent to 6 and 13

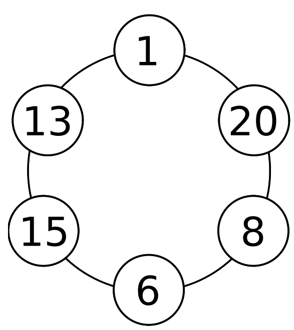
In this case, we can see from our table that 20 must be adjacent to 15 and 8, since 1 is no longer available. The two different ways to write such a circle are shown below.

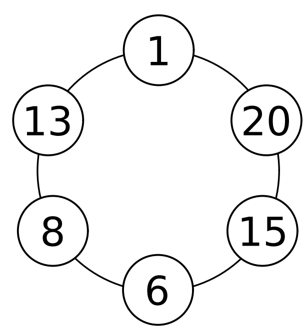




Case 3: 1 is adjacent to 13 and 20

In this case, we can see from our table that 6 must be adjacent to 15 and 8, since 1 is no longer available. The two different ways to write such a circle are shown below.





Therefore, we have found that there are 6 different arrangements. These are the arrangements shown in Cases 1, 2 and 3 above.