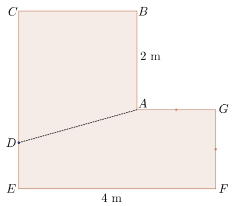


Problem of the Week

Problem D and Solution

It Must be Fair!

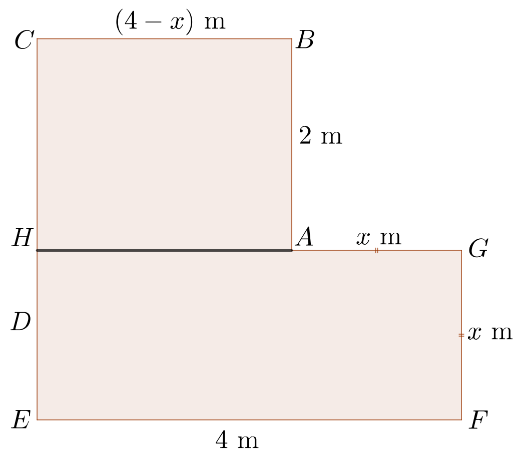
Problem

Two children share an L-shaped bedroom. They are constantly fighting over space. Their parents decide to temporarily partition the room with a curtain so that each child will have exactly the same area, in the hope that the space arguments will end. The layout of the room is represented by $ABCDEFG$ on the diagram. The room has square corners with $EF = 4$ m, $AB = 2$ m, and $AG = GF$. The area of the entire room is 11.2 m^2. The partitioning curtain is to be hung from point A to a point D on CE to divide the room into two parts of equal area. Where is D located on CE to accomplish the equal area split in order to make things fair?

Solution

Let x represent the length of AG. Since $GF = AG$, then $GF = x$.

Extend GA to intersect CE at H. This creates two rectangles $ABCH$ and $GHEF$ with $BC \parallel GH \parallel EF$. Then $BC = EF - AG = 4 - x$.



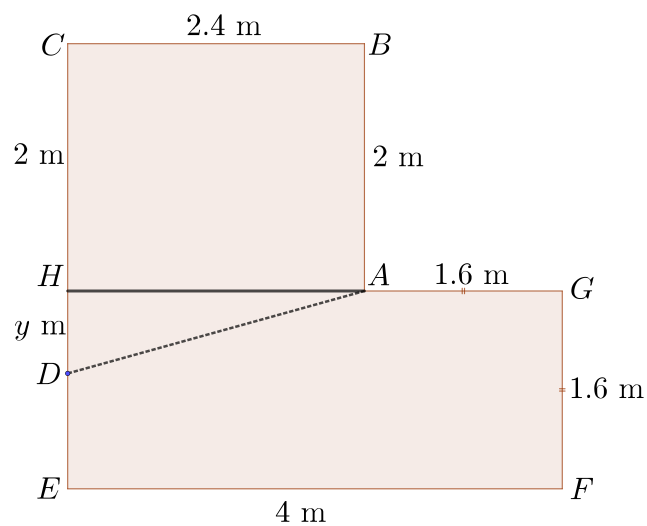
We can now find the value of x using the areas of rectangles.

$$\begin{aligned}
\text{Area } ABCDEFG &= \text{Area } ABCH + \text{Area } GHEF \\
11.2 &= (AB \times BC) + (GF \times EF) \\
11.2 &= 2(4 - x) + x(4) \\
11.2 &= 8 - 2x + 4x \\
3.2 &= 2x \\
1.6 &= x
\end{aligned}$$

Since $x = 1.6$ m, $AG = GF = 1.6$ m and $BC = 4 - x = 2.4$ m. Also, $HC = AB = 2$ m, and $EC = FG + AB = 1.6 + 2 = 3.6$ m.

Let y represent the length of DH.

A diagram with updated information is shown below.



$ABCD$ is a trapezoid with opposite parallel sides $AB = 2$ and $DC = 2 + y$. BC is perpendicular to both AB and DC, and $BC = 2.4$ m. We also know that the area of trapezoid $ABCD$ is half the area of $ABCDEFG$, so the area of trapezoid $ABCD$ is 5.6 m^2. Then,

$$\begin{aligned}
\text{Area of Trapezoid } ABCD &= \frac{BC \times (AB + DC)}{2} \\
5.6 &= \frac{2.4 \times (2 + 2 + y)}{2} \\
5.6 &= 1.2 \times (4 + y) \\
5.6 &= 4.8 + 1.2y \\
0.8 &= 1.2y \\
\frac{0.8}{1.2} &= y \\
\frac{8}{12} &= y \\
\frac{2}{3} &= y
\end{aligned}$$

Since $DC = 2 + y$, $DC = 2 + \frac{2}{3} = \frac{8}{3}$ m.

Also, since $ED = EC - DC$, $ED = 3.6 - \frac{8}{3} = \frac{18}{5} - \frac{8}{3} = \frac{54 - 40}{15} = \frac{14}{15}$ m.

The end of the partition located at D should be positioned $\frac{14}{15}$ m from E and $\frac{8}{3}$ m from C.