Problem

Three bags each contain tokens. The green bag contains 22 round green tokens, each with a different integer from 1 to 22. The red bag contains 15 triangular red tokens, each with a different integer from 1 to 15. The blue bag contains 10 square blue tokens, each with a different integer from 1 to 10.

Any token in a specific bag has the same chance of being selected as any other token from that same bag. There is a total of \(22 \times 15 \times 10 = 3300\) different combinations of tokens created by selecting one token from each bag. Note that selecting the 7 red token, the 5 blue token and 3 green token is different than selecting the 5 red token, 7 blue token and the 3 green token. The order of selection does not matter.

You select one token from each bag. What is the probability that two or more of the selected tokens have the number 5 on them?

Solution

Solution 1

There are 22 different numbers which can be chosen from the green bag, 15 different numbers which can be chosen from the red bag, and 10 different numbers which can be chosen from the blue bag. So there are a total of \(22 \times 15 \times 10 = 3300\) different combinations of numbers which can be produced by selecting one token from each bag.

To count the number of possibilities for a 5 to appear on at least two of the tokens, we will consider cases.

1. Each of the selected tokens has a 5 on it.
   
   This can only occur in 1 way.

2. A 5 appears on the green token and on the red token but not on the blue token.
   
   There are 9 choices for the blue token excluding the 5. A 5 can appear on the green token and on the red token but not on the blue token in 9 ways.

3. A 5 appears on the green token and on the blue token but not on the red token.
   
   There are 14 choices for the red token excluding the 5. A 5 can appear on the green token and on the blue token but not on the red token in 14 ways.

4. A 5 appears on the red token and on the blue token but not on the green token.
   
   There are 21 choices for the green token excluding the 5. A 5 can appear on the red token and on the blue token but not on the green token in 21 ways.

Summing the results from each of the cases, the total number of ways for a 5 to appear on at least two of the tokens is \(1 + 9 + 14 + 21 = 45\). The probability of 5 appearing on at least two of the tokens is \(\frac{45}{3300} = \frac{3}{220}\). 
Solution 2

This solution uses a known result from probability theory. If the probability of event \( A \) occurring is \( a \), the probability of event \( B \) occurring is \( b \), the probability of event \( C \) occurring is \( c \), and the results are not dependent on each other, then the probability of all three events happening is \( a \times b \times c \).

The probability of a specific number being selected from the green bag is \( \frac{1}{22} \) and the probability of any specific number not being selected from the green bag is \( \frac{21}{22} \).

The probability of a specific number being selected from the red bag is \( \frac{1}{15} \) and the probability of any specific number not being selected from the red bag is \( \frac{14}{15} \).

The probability of a specific number being selected from the blue bag is \( \frac{1}{10} \) and the probability of any specific number not being selected from the blue bag is \( \frac{9}{10} \).

In the following we will use \( P(p, q, r) \) to mean the probability of \( p \) being selected from the green bag, \( q \) being selected from the red bag, and \( r \) being selected from the blue bag. So, \( P(5, 5, \text{not } 5) \) means that we want the probability of a 5 being selected from the green bag, a 5 being selected from the red bag, and anything but a 5 being selected from the blue bag.

\[
\text{Probability of 5 being selected from at least two of the bags} = \text{Probability of 5 from each bag} + \text{Probability of 5 from exactly 2 bags}
\]

\[
= P(5, 5, 5) + P(5, 5, \text{not } 5) + P(5, \text{not } 5, 5) + P(\text{not } 5, 5, 5)
\]

\[
= \frac{1}{22} \times \frac{1}{15} \times \frac{1}{10} + \frac{1}{22} \times \frac{1}{15} \times \frac{9}{10} + \frac{1}{22} \times \frac{14}{15} \times \frac{1}{10} + \frac{21}{22} \times \frac{1}{15} \times \frac{1}{10}
\]

\[
= \frac{1}{3300} + \frac{9}{3300} + \frac{14}{3300} + \frac{21}{3300}
\]

\[
= \frac{45}{3300}
\]

\[
= \frac{3}{220}
\]

The probability of 5 appearing on at least two of the tokens is \( \frac{3}{220} \).