



Problem of the Week

Problem D and Solution

Go for the Gold



Problem

For orientation days at the University of Waterloo, various activities are available for incoming students to participate in. One activity is a simple game in which students reach into a box and randomly select a golf ball. The golf balls are all identical in size and are either black or gold. If a student selects a gold golf ball, then they win a prize. After a golf ball is selected, it is returned to the box. Initially, the box contained 300 gold golf balls and students had a 1 in 5 chance of selecting a gold golf ball. The organizers want to increase the chances of selecting a gold golf ball to 3 in 10. In order to do this, they add complete packages of golf balls. Each package of golf balls contains 60 golf balls, of which 65% are black. How many full packages of golf balls must be added to the box?

Solution

There were initially 300 gold golf balls in the box and the students had a 1 in 5 chance of winning. This means that there was a 4 in 5 chance of selecting a black golf ball. Therefore, there were four times as many black golf balls as gold golf balls. That is, there were $4 \times 300 = 1200$ black golf balls and a total of $300 + 1200 = 1500$ golf balls in the box before any new packages were added.

Each new package of golf balls contains 60 golf balls. Since 65% are black, there are $0.65 \times 60 = 39$ black golf balls and $60 - 39 = 21$ gold golf balls in each new package.

Let n represent the number of packages of golf balls added to the box to increase the chances of winning from 1 in 5 to 3 in 10. By adding n packages of golf balls to the box, we are adding $21n$ gold golf balls and $60n$ golf balls to the box. After adding the packages, the box will contain $300 + 21n$ gold golf balls and $1500 + 60n$ golf balls. Since the chances of winning are now 3 in 10,

$$\begin{aligned} \frac{\text{the number of gold golf balls}}{\text{the total number of golf balls}} &= \frac{3}{10} \\ \frac{300 + 21n}{1500 + 60n} &= \frac{3}{10} \\ 10(300 + 21n) &= 3(1500 + 60n) \\ 3000 + 210n &= 4500 + 180n \\ 30n &= 1500 \\ n &= 50 \end{aligned}$$

Therefore, 50 new packages of golf balls must be added to the box to raise the chances of winning from 1 in 5 to 3 in 10.

We can check this easily. After adding 50 new packages of golf balls, there would be $300 + 21(50) = 1350$ gold golf balls and a total of $1500 + 60(50) = 4500$ golf balls. Then, the ratio of gold golf balls to the total number of golf balls in the box is $\frac{1350}{4500} = \frac{3}{10}$, as required.

