Problem of the Week
Problem C and Solution
A Perfect Dozen

Problem

Twelve identical rectangles are arranged as shown in the diagram above to form a large rectangle \(PQRS\). If the area of rectangle \(PQRS\) is 540 cm\(^2\), determine the dimensions of the smaller rectangles.

Solution

Solution 1

Let \(x\) be the width of one of the smaller identical rectangles, in cm. Five of the smaller rectangles are stacked on top of each other creating \(PS\), so \(PS = x + x + x + x + x = 5x\). Since \(PQRS\) is a rectangle, \(PS = QR = 5x\). But \(5x\) then also is the length of a smaller rectangle. Therefore, a smaller rectangle is \(5x\) cm by \(x\) cm. This information is all marked on the following diagram.

The area of rectangle \(PQRS\) is the same as 12 times the area of one of the smaller rectangles.

\[
\text{Area } PQRS = 12 \times \text{Area of one smaller rectangle}
\]

\[
540 = 12 \times 5x \times x
\]

\[
540 = 60 \times x^2
\]

Dividing both sides by 60, we obtain \(x^2 = 9\) and \(x = 3\) follows. (\(x > 0\) since \(x\) is the width of a smaller rectangle.)

The width of a smaller rectangle is \(x = 3\) cm and the length of a smaller rectangle is \(5x = 5(3) = 15\) cm.

Therefore, the smaller rectangles are each 15 cm long and 3 cm wide.
Solution 2

Let $x$ be the width of one of the smaller identical rectangles, in cm. Five rectangles are stacked on top of each other creating $PS$, so $PS = x + x + x + x + x = 5x$.

Also $PQRS$ is a rectangle so $QR = PS = 5x$ and $5x$ is also the length of a smaller rectangle.

Now $SR$ is made up of the lengths of two of the smaller rectangles plus the widths of two of the smaller rectangles. Therefore, $SR = 5x + 5x + x + x = 12x$ and rectangle $PQRS$ is $12x$ cm long and $5x$ cm wide.

To find the area of $PQRS$ we multiply the length $SR$ by the width $QR$.

\[
\text{Area } PQRS = SR \times QR = 12x \times 5x = 60x^2
\]

Dividing by 60, we obtain $x^2 = 9$ and $x = 3$ follows. ($x > 0$ since $x$ is the width of a smaller rectangle.)

The width of a smaller rectangle is $x = 3$ cm and the length of a smaller rectangle is $5x = 5(3) = 15$ cm.

Therefore, the smaller rectangles are each 15 cm long and 3 cm wide.