Problem of the Week
Problem C and Solution
A Missing Length

Problem

A circle with center \( B \) and radius 13 cm has three distinct points, \( F, D \) and \( E \), on its circumference so that \( BF \perp BE \) and \( D \) is on the minor arc \( FE \). Point \( A \) is on \( BF \) so that \( DA \perp BF \). The point \( C \) is on \( BE \) so that \( ABCD \) is a rectangle and the distance from \( C \) to \( E \) is 1 cm. Determine the distance from \( A \) to \( F \).

Solution

Construct radius \( BD \).

Since the radius of the circle is 13 cm, \( BF = BD = BE = 13 \) cm.

Then \( BC = BE - CE = 13 - 1 = 12 \) cm.

Since \( ABCD \) is a rectangle, \( \angle BCD = 90^\circ \).

Using the Pythagorean Theorem in right \( \triangle BCD \),

\[
DC^2 = DB^2 - BC^2
= 13^2 - 12^2
= 169 - 144
= 25
\]

\( DC = 5 \) cm (since \( DC > 0 \))

Since \( ABCD \) is a rectangle, \( AB = DC = 5 \) cm.

Then \( AF = BF - AB = 13 - 5 = 8 \) cm.

Therefore, the length of \( AF \) is 8 cm.