Problem of the Week
Problem B and Solution
Paving the Way

Problem
Munit’s driveway is 8 m wide and 16 m long. He wants to cover the entire rectangular area with pavers which are only available in sets of 10 pavers. Each set of 10 pavers covers 3 square metres in total.

a) If each set of 10 pavers costs $100, how much will it cost Munit to buy enough sets of pavers for the job? Add 1 extra set of pavers to your purchase to allow for possible breakage.

b) Each paver in a set is $\frac{1}{2}$ m by $\frac{3}{5}$ m. Make a plan for placing the pavers so that they completely cover the driveway. In your design, how many pavers will need to be cut in order to fit the driveway? How much will be cut from each of these pavers?
You may assume the pavers can be laid either horizontally or vertically.
There are many ways to place the pavers. Compare your design to the design of others in your class.

Solution
a) The total area of pavers required is $8 \text{ m} \times 16 \text{ m} = 128 \text{ m}^2$. Since each set covers $3 \text{ m}^2$, Munit will need $128 \div 3 = 42\frac{2}{3} \approx 42.7$, so 43 sets of pavers. If an extra set of pavers is purchased, Munit will need 44 sets. At a cost of $100 per set, his total cost is thus $100 \times 44 = $4400.

b) Each paver is 50 cm wide by 60 cm long ($\frac{1}{2}$ m by $\frac{3}{5}$ m).
We present three of many possible plans.

PLAN 1:
Set the pavers vertically across the 8 m = 800 cm width of the driveway. This requires $800 \div 50 = 16$ pavers. The 16 m = 1600 cm length of the driveway would need $1600 \div 60 = 26\frac{2}{3}$ pavers. So the final 27th row would need each of the 16 pavers cut to only $\frac{2}{3} \times 60 = 40$ cm, i.e., 16 cuts are required. That is, 20 cm is cut off of each of the 16 pavers in the 27th row. This method would use $27 \times 16 = 432$ pavers. Since we purchased an extra set of pavers, we have enough.
Plan 2:
Alternate rows of pavers laid vertically and horizontally, starting with a vertical row. This will require exactly 15 vertical rows and 14 horizontal rows to cover the length of the driveway, since \(15 \times 60 + 14 \times 50 = 900 + 700 = 1600\) cm. Further, for the vertical rows, the width will require 16 pavers, since \(800 \div 50 = 16\). However, the horizontal rows will require 14 pavers, since \(800 \div 60 = 13\frac{1}{3}\). The pavers at the ends of these 14 horizontal rows will need to be cut to \(\frac{1}{3}\) of 60 = 20 cm by 50 cm, requiring 14 cuts. That is, 40 cm is cut off of each of the pavers at the ends of the 14 horizontal rows. Each row with pavers laid vertically has 16 pavers and there are 15 such rows, so there are \(15 \times 16 = 240\) pavers laid vertically. Each row with pavers laid horizontally has 14 pavers and there are 14 such rows, so there are \(14 \times 14 = 196\) pavers laid horizontally. This plan uses \(240 + 196 = 436\) pavers. Since we purchased an extra set of pavers, we have enough to complete the job.

Plan 3:
Lay 5 pavers horizontally, followed by 4 pavers vertically, and then 5 more pavers horizontally. Do this for a total of 5 rows. This creates the unshaded part in the diagram to the upper right. Now, add 10 more horizontal pavers to complete the rectangle. These last 10 pavers are shaded in the diagram. The large rectangle is 8 m by 3 m. (You can verify this.) Do this process a total of 5 times to cover 8 m by 15 m of the driveway.

Now, a strip 8 m by 1 m is needed to complete the coverage. This small strip is shown in the diagram to the bottom right.

In this small strip, 4 of the vertically placed pavers extend 0.2 m or 20 cm past the end of the driveway. Therefore, 4 cuts of 20 cm each are required. Each of the 8 m by 3 m sections has 80 pavers. There are 5 of these sections, so there are \(5 \times 80 = 400\) pavers used. In the final two rows, 28 pavers are used. Therefore, the total number of pavers used is \(400 + 28 = 428\). Since we have 440 pavers, we have 12 extra pavers. We could return the extra set to the store and reduce our cost to $4300.