Problem of the Week
Problem A and Solution
Pyramid Nets

Problem
Draw a diagram of each of the nets that can be used to construct a square based pyramid.

Solution
We will define two nets as congruent (i.e. the same) if you can rotate and/or flip one of the nets to make it identical to the other. With this restriction, there are a total of eight nets that can be used to create a square based pyramid. The eight nets are shown below.
Teacher’s Notes

As mathematicians, we should convince ourselves that there are no more possible nets for the pyramid. It would take quite a bit of writing to make a full argument, but let’s at least think carefully about how to count the possible nets. Let’s start by thinking about how we start to form each net. Any possible net would have at least one, and at most four triangles sharing a side with the square. In the cases of one, three, or four triangles, ignoring any congruent nets, there is only one possible arrangement in each case as we see here:

In the case where all four triangles share a side with the square, this is the one and only net possible:

In the case where three triangles share a side with the square, it is not possible to form a pyramid if we put the fourth triangle between two other triangles. If we try to, we will see the triangles get in the way of each other. So the only possible arrangement is to add the fourth triangle on the outside edge of one of the original triangles as we see with this net:
In the case where one triangle shares a side with the square, there are two cases to consider. We can arrange the other three triangles together and then add them to one side of the original triangle. Most arrangements will not work as the triangles will get in the way of each other as we try to form the pyramid. However, one arrangement will work as we see in this net:

The other possibility is to have one of the additional triangles on one side and two additional triangles on the other side of the original triangle. Again, if we try it, there is only one arrangement of this kind that will work as we see in this net:

There is one more case to consider as a starting point. This is the case where we start with two triangles that share sides with the square. There are two starting arrangements: the triangles can be on opposite sides of the square or they can be on adjacent sides of the square as we see here:

With each of these starting arrangements, through trial and error we can show that there are only two possible arrangements of the additional two triangles that will correctly form a net. These lead to the last four nets as we see here: