Problem of the Week
Problem D and Solution
Green Thumbs

Problem
Two local gardening enthusiasts, Mia Gardner and Ivana Grow, competed for a contract to prepare the gardens at a local park. The contract was awarded to Mia. She worked 9 hours a day for 15 days, and was able to complete \( \frac{3}{8} \) of the entire job. The Parks Commission wanted to move the job along to complete it in a shorter period of time, so the Commission then hired Ivana Grow to work with Mia. Together they completed the remainder of the job in another 10 days, each working 9 hours per day. If Ivana had been hired originally instead of Mia, how many hours would it have taken her to complete the entire job on her own?

Solution
We must make some reasonable assumptions. Each gardener worked at a constant rate each hour, every day. These rates may or may not have been the same for the two gardeners.

Since Mia completed \( \frac{3}{8} \) of the job in 15 days, she would complete \( \frac{1}{3} \) of \( \frac{3}{8} \) or \( \frac{1}{8} \) of the job in 5 days.

Since Mia had completed \( \frac{3}{8} \) of the job when Ivana started to work, \( \frac{5}{8} \) of the job was left to be completed. Together they completed \( \frac{5}{8} \) of the job in 10 days. Since Mia can complete \( \frac{1}{8} \) of the job in 5 days, she would have completed \( \frac{2}{8} \) of the job in these 10 days. Therefore, Ivana completed the remaining \( \frac{5}{8} - \frac{2}{8} = \frac{3}{8} \) of the job in these 10 days.

Since Ivana worked 9 hours a day, this means she completed \( \frac{3}{8} \) of the job in \( 10 \times 9 = 90 \) hours. Therefore, she completed \( \frac{1}{8} \) of the job in 30 hours. Therefore, she could have completed the entire job on her own in \( 8 \times 30 = 240 \) hours.

For Your Information
Mia completed \( \frac{1}{8} \) of the job in 5 days. The whole job could be completed by Mia in \( 8 \times 5 = 40 \) days or 360 hours.

As it was, Mia worked a total of 25 days at 9 hours per day and Ivana worked 10 days at 9 hours per day. They worked a total of \( 25 \times 9 + 10 \times 9 = 315 \) hours.

We know that together, Mia and Ivana completed \( \frac{5}{8} \) of the job in 10 days. Then, in 2 days they would have completed \( \frac{1}{8} \) of the job and in 16 days they would have completed the entire job. That is, in \( 16 \times 9 = 144 \) hours, working together from the start they would have completed the job.