Problem of the Week
Problem D
ELEVEN

The word ELEVEN contains four different letters, $E$, $L$, $V$, and $N$. Each letter in the word ELEVEN is assigned a different integer value between 0 and 9, inclusive, to create a six-digit positive integer. If, for example, $E = 4$, $L = 5$, $V = 6$, and $N = 1$, then the resulting number is 454641. The choice of these digit values for the letters in the word ELEVEN is particularly interesting since the resulting number is divisible by 11.

1. Determine the values of $E$, $L$, $V$, and $N$ which make ELEVEN as large as possible subject to the condition that ELEVEN must be divisible by 11; and

2. determine the values of $E$, $L$, $V$, and $N$ which make ELEVEN as small as possible, subject to the condition that ELEVEN must be divisible by 11. (Also remember, $E \neq L \neq V \neq N$.)

Did you know that a number is divisible by 11 exactly when the sum of the digits in the odd digit positions minus the sum of the digits in the even digit positions is divisible by 11? The number 454641 is divisible by 11 since $(4 + 4 + 4) - (5 + 6 + 1) = 12 - 12 = 0$ and 0 is divisible by 11. The number 282623 is also divisible by 11 since $(2 + 2 + 2) - (8 + 6 + 3) = 6 - 17 = -11$ and $-11$ is divisible by 11.