Problem of the Week
Problem D and Solution
Cut Along the Dotted Line

Problem

Four pieces of lumber are placed in parallel positions, as shown above, perpendicular to the line $W$.

- Piece $A$ is 5 m long and touches $W$
- Piece $B$ is 3 m long and its left end is 3 m from the line $W$
- Piece $C$ is 5 m long and its left end is 2 m from the line $W$
- Piece $D$ is 4 m long and its left end is 1.5 m from the line $W$

A single cut, parallel to $W$, is made along the dotted line $L$. The total length of lumber on each side of $L$ is the same. What is the length, in m, of the part of $A$ to the left of the cut?

Solution

Solution 1

Suppose that the distance from line $W$ to line $L$ is $d$ m. Therefore, the total length of piece $A$ to the left of the cut is $d$ m.

Since piece $B$ is 3 m from line $W$, then the length of piece $B$ to the left of $L$ is $(d - 3)$ m.

Similarly, the lengths of pieces $C$ and $D$ to the left of line $L$ are $(d - 2)$ m and $(d - 1.5)$ m respectively.

Therefore, the total length of lumber to the left of line $L$ is

$$d + (d - 3) + (d - 2) + (d - 1.5) = 4d - 6.5 \text{ m}.$$ 

Since the total length of lumber on each side of the cut is equal, then the length on the left side is also $\frac{1}{2}(5 + 3 + 5 + 4) = 8.5 \text{ m}$.

Therefore, $4d - 6.5 = 8.5$, or $4d = 15$, or $d = 3.75$. Therefore, the length of the part of piece $A$ to the left of $L$ is 3.75 m.
Solution 2

Suppose that the distance from line $W$ to line $L$ is $d$ m. Therefore, the total length of piece $A$ to the left of the cut is $d$ m. Since $A$ is 5 m long, the length of $A$ to the right of the cut is $(5 - d)$ m.

Since piece $B$ is 3 m from line $W$, then the length of piece $B$ to the left of $L$ is $(d - 3)$ m. Since $B$ is 3 m long, the length of $B$ to the right of the cut is $3 - (d - 3)$, or $(6 - d)$ m.

Since piece $C$ is 2 m from line $W$, then the length of piece $C$ to the left of $L$ is $(d - 2)$ m. Since $C$ is 5 m long, the length of $C$ to the right of the cut is $5 - (d - 2)$, or $(7 - d)$ m.

Since piece $D$ is 1.5 m from line $W$, then the length of piece $D$ to the left of $L$ is $(d - 1.5)$ m. Since $D$ is 4 m long, the length of $D$ to the right of the cut is $4 - (d - 1.5)$, or $(5.5 - d)$ m.

Therefore, the total length of lumber to the left of line $L$ is
\[d + (d - 3) + (d - 2) + (d - 1.5) = 4d - 6.5\] m.

The total length of lumber to the right of line $L$ is
\[(5 - d) + (6 - d) + (7 - d) + (5.5 - d) = 23.5 - 4d\] m.

Since the total length of lumber on each side of the cut is equal, then
\[4d - 6.5 = 23.5 - 4d\]
\[8d = 30\]
\[d = 3.75\]

Therefore, the length of the part of piece $A$ to the left of $L$ is 3.75 m.