Problem of the Week
Problem C and Solution
Garden Paths

Problem

In a garden, Jazzmin travels through the labyrinth shown above from Entrance to Exit. She is only allowed to travel east, south, or southeast along a path. How many different routes can Jazzmin take from Entrance to Exit?

Solution

We could solve this problem by tracing out different routes and counting how many we find. We will set up a systematic approach to doing so, to ensure that we do not miss any routes.

We begin by labelling the Entrance with the letter $S$ for Start and the Exit with the letter $F$ for Finish. We will then label the seven intersections in the maze as $A, B, C, D, E, G,$ and $H$, as shown.

Since Jazzmin can only travel east, south, or southeast along a path, starting at $S$ she has only two choices as to where to go next: $A$ or $C$.

**Case 1:** Jazzmin travels from $S$ to $A$

Again, since Jazzmin can only travel east, south, or southeast along a path, she has only two choices as to where to go next: $B$ or $D$.

If Jazzmin travels to $B$, then since she can only travel east, south or southeast, she must go to $E$ next, followed by $F$. Therefore, one route from $S$ to $F$ is $SABEF$.

If Jazzmin travels to $D$, then since she can travel east, south or southeast, she can go to $E, F$ or $H$ next.

- If she travels from $D$ to $E$, she must then go to $F$. Therefore, one route from $S$ to $F$ is $SADEF$.
- If she travels from $D$ to $F$, we have found another route. Therefore, one route from $S$ to $F$ is $SADF$.
- If she travels from $D$ to $H$, she must then go to $F$. Therefore, one route from $S$ to $F$ is $SADHF$.

Therefore, there are four routes from $S$ to $F$ in which Jazzmin first travels from $S$ to $A$. They are $SABEF, SADEF, SADF,$ and $SADHF$. 
**Case 2:** Jazzmin travels from $S$ to $C$

Since Jazzmin can travel east, south or southeast, she could travel to $D$, $H$ or $G$ next.

- If she travels from $C$ to $D$, she again has 3 choices: travel from $D$ to $E$, $F$ or $H$. We get three different routes: $SCDEF$, $SCDF$ and $SCDHF$.
- If she travels from $C$ to $H$, from $H$ she must go to $F$. We have found another route, $SCHF$.
- If she travels from $C$ to $G$, she must then go to $H$ then $F$. We have found another route, $SCGHF$.

Therefore, there are five routes from $S$ to $F$ in which Jazzmin first travels from $S$ to $C$. They are $SCDEF$, $SCDF$, $SCDHF$, $SCHF$, and $SCGHF$.

In total, there are $4 + 5 = 9$ different routes that Jazzmin can take from Entrance to Exit.