Problem of the Week
Problem B and Solution
A Round Trunk in a Square Hole

Problem

Rhab's mother and father decided that they wanted him to spend more time outdoors, so they built him a tree fort with a red maple tree growing right through the middle of the fort.

a) Rhab's mom found two pieces of plywood with dimensions 1.2 m (120 cm) by 2.4 m (240 cm) to use for the floor. What will be the total floor area of the tree fort if they use both sheets of plywood to form a square floor for the tree fort?

b) Rhab’s parents thought about how they should make the hole in the plywood floor so that the tree would fit. They decided that a square would be the easiest solution. Rhab’s dad measured the distance around the tree (the circumference) and found that it was 140 cm. He was about to cut a square with side length 35 cm when Rhab, remembering what he learned in school, stopped his dad before it was too late. Why did Rhab stop his dad?

c) Rhab found the diameter of the tree to be about 45 cm. If they decide to cut a square hole through which the trunk will fit, what are the dimensions and area of the square hole that his dad should cut?

d) What is the remaining area of the floor of the tree fort after the square hole has been cut out?

e) Where Rhab’s family lives, red maples grow to around 60 cm in diameter. How might this affect the design of the tree fort?

Solution

a) By putting the two pieces of plywood together, the floor of the treehouse will be 2.4 m × 2.4 m = 5.76 m².

b) The distance around the tree does not help you determine the area taken up by the tree or the dimensions of the sides of the square to be cut. Rhab’s father would need to find the radius or diameter of the tree. The diameter of the tree can be found by dividing 140 cm (the known circumference) by \( \pi \) (approximately 3.14).

c) If they decide to cut the square with side lengths of 45 cm, then the area will be 45 cm × 45 cm = 2025 cm² = 0.2025 m². If they decide to cut the square slightly larger, say with side lengths of 50 cm, then the area will be 50 cm × 50 cm = 2500 cm² = 0.25 m².

**COMMENT:** If they made the hole 50 cm × 50 cm, that would allow for 5 cm of growth in the diameter of the tree. A maple tree’s circumference grows by approximately 1.5 cm each year. By the time the tree has grown an extra 5 cm in diameter, Rhab may be too old to be hanging around in his tree fort.

d) After removing a 50 cm × 50 cm square, the remaining area would be 5.76 m² − 0.25 m² = 5.51 m².

e) If the diameter of the red maple tree can grow to 60 cm, then they should probably make the side lengths of the square about 65 cm. Then 65 cm × 65 cm = 4225 cm² = 0.4225 m² of space is taken from the fort, leaving 5.76 m² − 0.4225 m² = 5.3375 m² of space for frolicking. The area would only be reduced by about 0.2 m².