Problem of the Week
Problem B and Solution
Not So Phunee...

Problem
Phunee Ghai is a struggling stand-up comedian who has comedy gigs around town on five nights out of seven. He tries to wear different outfits, just in case some audience members may want to see him more than once. Because Phunee doesn’t have a lot of money, he tries to strategically coordinate his outfits so that he does not have to repeat his outfits. Here’s what Phunee has in his closet.

- 1 flannel yellow and green shirt
- 1 pair Spikee running shoes
- 1 pair blue jeans
- 1 pair cowboy boots
- 1 pair khaki chinos (pants)
- 1 pair orange shorts
- 1 retro cable-knit baby blue sweater
- 2 long-sleeved T-shirts (one navy blue, one light green)
- 1 pair velcro-strapped loafers
- 1 pair black jeans

a) Assuming he works every night, for how many weeks can Phunee do his stand-up comedy routine, wearing a different combination of clothing each time, without repeating an outfit?

b) What is the probability of him wearing a pair of jeans and a sweater?

c) Actually, Phunee does not work on Sundays and Mondays. If he started wearing his combinations on a Thursday, on what day would he have to make sure he has a new article of clothing to add to his wardrobe?
Solution

a) The diagram shows the start of a tree structure for enumerating Phunee’s possible outfits.

For each shirt, there are four possible pairs of pants. For each of those, there are three possible pairs of shoes. Thus, for each shirt, there are $4 \times 3 = 12$ possibilities. Since there are 4 different shirts, in total there are $12 \times 4 = 48$ possible different outfits.

One could also multiply the number of shirts by the number of shoes and by the number of pants, giving $4 \times 3 \times 4 = 48$ outfits. Therefore, Phunee could work 6 weeks and 6 days before having to wear the same outfit twice.

b) Phunee has 1 sweater, 2 pairs of jeans (blue jeans and black jeans), and 3 different pairs of shoes (loafers, runners, boots). We list all possible combinations:

<table>
<thead>
<tr>
<th>Sweater</th>
<th>Blue Jeans</th>
<th>Loafers</th>
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<tbody>
<tr>
<td>Sweater</td>
<td>Blue Jeans</td>
<td>Runners</td>
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<tr>
<td>Sweater</td>
<td>Blue Jeans</td>
<td>Boots</td>
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<tr>
<td>Sweater</td>
<td>Black Jeans</td>
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There are 6 combinations out of a total of 48 possible combinations, so the probability of wearing a sweater and jeans is $\frac{6}{48} = \frac{1}{8}$.

Another way to look at this would be to calculate the probability of wearing a sweater and the probability of wearing jeans. Phunee has only 1 sweater out of 4 shirts, so his chances of wearing a sweater are $\frac{1}{4}$. He has 2 pairs of jeans out of 4 pairs of pants, so his chances of wearing jeans are $\frac{2}{4} = \frac{1}{2}$. Since these are independent events, the probability of him wearing a pair of jeans and a sweater is $\frac{1}{4} \times \frac{2}{4} = \frac{2}{16} = \frac{1}{8}$.

Some of the terminology and theory used in this second approach to part (b) will be developed in later mathematics courses.

c) Since Phunee works 5 days out of 7, 9 weeks of 5 working days would be 45 days. So three days into his 10th week of work, Phunee will have worn each of his 48 outfits once, and will need to buy something else to complement his wardrobe. Since he started on a Thursday, this would normally occur on a Sunday. Since Phunee doesn’t work Sunday or Monday, he will need a new article of clothing on his 49th day of work, which is a Tuesday.