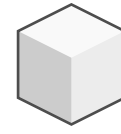




Problem of the Week

Problem E and Solution

Sweet!



Problem

A bowl contained 320 grams of pure white sugar. Mixture Y was formed by taking x grams of the white sugar out of the bowl, adding x grams of brown sugar to the bowl, and then mixing uniformly. In Mixture Y, the ratio of the mass of the white sugar to the mass of the brown sugar, expressed in lowest terms, was $w : b$. Mixture Z is formed by taking x grams of Mixture Y out of the bowl, adding x grams of brown sugar to the bowl, and then mixing uniformly. In Mixture Z, the ratio of the mass of the white sugar to the mass of the brown sugar is $49 : 15$. What is the value of $x + w + b$?

Solution

Initially, the bowl contained 320 g of white sugar and 0 g of brown sugar. Mixture Y contained $(320 - x)$ g of white sugar and x g of brown sugar. When Mixture Z (the final mixture) is formed, there is still 320 g of sugar in the bowl.

In Mixture Z, the ratio of the mass of white sugar to the mass of brown sugar is $49 : 15$. Therefore, the mass of white sugar is $\frac{49}{49+15} \cdot 320 = \frac{49}{64} \cdot 320 = 49 \cdot 5 = 245$ g and the mass of brown sugar in Mixture Z is $320 - 245 = 75$ g.

Mixture Y consisted of $(320 - x)$ g of white sugar and x g of brown sugar, which were thoroughly mixed together. Therefore, each gram of Mixture Y consisted of $\frac{320-x}{320}$ g of white sugar and $\frac{x}{320}$ g of brown sugar.

To form Mixture Z, x g of Mixture Y were removed. This amount of Mixture Y that was removed contained $x \cdot \frac{x}{320} = \frac{x^2}{320}$ g of brown sugar. Therefore, the mass of brown sugar in Mixture Z is the original x g added to get Mixture Y minus the $\frac{x^2}{320}$ g from mixture Y plus the new x g making $x - \frac{x^2}{320} + x = 2x - \frac{x^2}{320}$ g of brown sugar.

But we determined earlier that Mixture Z contains 75 g of brown sugar. Therefore,

$$\begin{aligned} 2x - \frac{x^2}{320} &= 75 \\ 0 &= x^2 - 2(320)x + 75(320) \\ 0 &= x^2 - 640x + 24000 \\ 0 &= (x - 40)(x - 600) \end{aligned}$$

Therefore, $x = 40$ or $x = 600$. Since the initial mixture consists of 320 g of sugar, then $x < 320$. It follows that $x = 40$. Then Mixture Y consists of $320 - 40 = 280$ g of white sugar and 40 g of brown sugar. The ratio of these masses is $280 : 40$ or $7 : 1$ in lowest terms. Thus, $w = 7$ and $b = 1$. Therefore, $x + w + b = 40 + 7 + 1 = 48$.

