Problem of the Week
Problem E and Solution
No Repeat

Problem

When $\dfrac{1}{50^{2018}}$ is written as a decimal, it terminates.

What is the last non-zero digit in the decimal representation of $\dfrac{1}{50^{2018}}$?

Solution

Our first instinct might be to use our calculator to get an idea of how the last digit behaves for the first few powers of $\frac{1}{50}$. Most calculators let us down too quickly.

Notice that $\dfrac{1}{50^{2018}} = \left(\dfrac{1}{50}\right)^{2018} = \left(\dfrac{1}{100} \times 2\right)^{2018} = (0.01 \times 2)^{2018} = (0.01)^{2018} \times 2^{2018}$

The last non-zero digit in the decimal representation of $\dfrac{1}{50^{2018}}$ will therefore be the last non-zero digit in the decimal representation of $(0.01)^{2018}$ multiplied by the last digit of 2^{2018}.

Since the last non-zero digit in the decimal representation of $(0.01)^{2018}$ is 1, the last non-zero digit in the decimal representation of $\dfrac{1}{50^{2018}}$ will therefore be the last digit of 2^{2018}.

We now examine the last digit of various powers of 2:

$$\begin{aligned}
2^1 &= \mathbf{2}\\
2^2 &= \mathbf{4}\\
2^3 &= \mathbf{8}\\
2^4 &= 1\mathbf{6}\\
2^5 &= 3\mathbf{2}\\
2^6 &= 6\mathbf{4}\\
2^7 &= 12\mathbf{8}\\
2^8 &= 25\mathbf{6}
\end{aligned}$$

Notice that the last digit repeats every four powers of 2. This pattern continues. 2^9 ends with a 2, 2^{10} ends with a 4, 2^{11} ends with an 8, 2^{12} ends with a 6, and so on.

We need to determine the number of complete cycles in 2018.

$$\frac{2018}{4} = 504\frac{1}{2}$$

Therefore, there are 504 complete cycles. Since $504 \times 4 = 2016$, this means 2^{2016} ends with a 6, 2^{2017} ends with a 2, and 2^{2018} ends with a 4.

Since 2^{2018} ends with a 4, $\dfrac{1}{50^{2018}}$ also ends with a 4.